HANDBOOK
OF
ENGINEERING MATHEMATICS

BY

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AUTHORS' PREFACE

In the present edition, the handbook has been revised to include a number of additions to the mathematical sections and to the tables of mathematical functions, and the values of physical and chemical constants have been revised to agree with recent investigation.

The authors are especially indebted to Professor W. D. Ennis of the United States Naval Academy, Annapolis, for a critical reading of the revised manuscript and for valuable suggestions; and for the section on aeronautics which has been contributed by Professor Ennis; also to Professor Ernst J. Berg and John N. Vedder of Union College for advice in connection with certain sections.

The authors wish to express their thanks to Professors Irving P. Church, G. A. Goodenough, and William A. Granville, who have kindly given permission for the use of special material, tables, and constants from their works, and to whom proper credit is given where such material appears.

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Engineering Mathematics

ALGEBRA

Quadratic Equations

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2\,a}$$

The term $b^2 - 4\,ac$, called the **discriminant,** determines the nature of the roots. If b^2 is greater than $4\,ac$, the roots are real. If b^2 is less than $4\,ac$, the roots are imaginary. And if $b^2 = 4\,ac$, the roots are real and equal.

Exponents

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$a^m = \frac{1}{a^{-m}} \qquad a^{-m} = \frac{1}{a^m}$$

$$(a^m)^n = a^{mn}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^{\frac{1}{mn}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$$

$$\sqrt[n]{a^n} = a^{\frac{n}{n}} = a$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b} = a^{\frac{1}{n}} b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$$

$$\left(\frac{a}{b}\right)^{\frac{1}{m}} = \sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$$

Special and Indeterminate Forms

$$a^0 = 1$$

$$a^\infty = \infty, \qquad a > 1$$

$$a^{-\infty} = \frac{1}{a^\infty} = \frac{1}{\infty} = 0, \qquad a > 1$$

$$\frac{a}{0} = \infty \qquad \frac{a}{\infty} = 0$$

$$\frac{\infty}{a} = \infty \qquad \frac{0}{a} = 0$$

$0 \cdot \infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, 1^∞, ∞^0, $\infty - \infty$ are indeterminate.

For the evaluation of indeterminate forms, see page 50.

Binomial Theorem

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Proportion

If
$$a : b = c : d \quad \text{or} \quad \frac{a}{b} = \frac{c}{d}$$

then
$$ad = bc \qquad \frac{a}{c} = \frac{b}{d} = \frac{a+b}{c+d} = \frac{a-b}{c-d}$$

$$\frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c} = \frac{b-d}{a-c}$$

If
$$\frac{a}{b} = \frac{c}{d} \quad \text{and} \quad \frac{e}{f} = \frac{g}{h}$$

then

$$\frac{ae}{bf} = \frac{cg}{dh} \quad \text{and} \quad \frac{ag}{bh} = \frac{ce}{df}$$

If

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

then

$$\frac{a+c+e}{b+d+f} = \frac{ma+nc+pe}{mb+nd+pf} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

Arithmetical Progression

An **arithmetical progression** is one whose terms increase or decrease by a common difference,

$$a,\ a+d,\ a+2\,d,\ a+3\,d,\ \ldots$$

the **last term** is $L = a + (n-1)\,d$

the **sum of the terms** is

$$S = \frac{n}{2}(a+L) = \frac{n}{2}[2\,a + (n-1)\,d]$$

a = first term

n = number of terms

d = common difference

Geometrical Progression

Quantities are in **geometrical progression** when each term is equal to the preceding term multiplied by a constant,

$$a,\ ar,\ ar^2,\ ar^3,\ \ldots$$

the **last term** is $L = ar^{n-1}$

the **sum of the terms** is

$$S = \frac{a\,(r^n-1)}{r-1} = \frac{a\,(1-r^n)}{1-r} = \frac{rL-a}{r-1}$$

a = first term

r = constant ratio

n = number of terms

The **sum** of an **infinite number** of terms in geometrical progression is

$$S = \frac{a}{1-r}$$

in which the ratio r must be less than 1 if the series is to be convergent (see Infinite Series).

Logarithms

The **logarithm** of any number to a given base is the power to which the base must be raised in order to produce the given number, thus:

if $\quad x^m = y, \quad$ then $\quad m = \log_x y,$

that is, m is the logarithm of y to the base x.

The following relations hold for any base:

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log a^n = n \log a$$

$$\log \frac{1}{a} = -\log a$$

The **base of the common system** of logarithms is 10.

The **base of the natural system** of logarithms (also called Naperian or hyperbolic logarithms) is $e = 2.7182818284 \ldots.$

A logarithm may be transformed from any given base to any other desired base by the relation:

$$\log_b N = \frac{\log_a N}{\log_a b}.$$

To transform a logarithm **from base 10 to base e,**

multiply by 2.302585 . . . (where 2.302585 . . . is the logarithm of 10 to the base e):

$$\log_e a = 2.302585 \log_{10} a$$

To transform a logarithm **from base e to base 10,** divide by 2.302585:

$$\log_{10} a = \frac{1}{2.302585} \log_e a = 0.434294 \log_e a$$

Special forms:

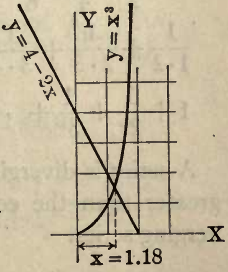
$$\log 1 = 0 \qquad \text{(to any base)}$$
$$\log_a a = 1 \qquad \log_e e = 1$$
$$\log_b 0 = -\infty \qquad \log_b \infty = \infty$$
$$b > 1$$

Cubic and Higher Degree Equations

The approximate values of the real roots of an algebraic equation containing only one variable may be found graphically.

For instance, let it be required to solve the equation $x^3 + Ax - B = 0$. This may be written as $x^3 = -Ax + B$, or as two simultaneous equations $y = x^3$ and $y = -Ax + B$. The graph of each of these equations being plotted, the abscissas of their points of intersection give the real roots of the cubic. The curve $y = x^3$ should be plotted on cross-section paper by the aid of a table of cubes. The curve $y = -Ax + B$ is the equation of a straight line, and is therefore determined by plotting two points.

Illustrative Example. Solve the equation $x^3 + 2x - 4 = 0$ graphically. Write the equation in the form $x^3 = 4 - 2x = y$ and plot the equations $y = x^3$ and $y = 4 - 2x$. Their intersection gives the solution $x = 1.18$.



Algebraic equations of any degree may be solved by Newton's method of approximation; see page 51.

Transcendental Equations

The graphic method given under Cubic and Higher Degree Equations is also applicable to many transcendental equations. Thus, the equation $Ax - \sin x = 0$ may be solved by plotting the two simultaneous equations $y = Ax$ and $y = \sin x$. The curve $y = \sin x$ is readily plotted with the aid of a table of sines, while the other curve $y = Ax$ is a straight line passing through the origin.

Infinite Series

An **infinite series** is one containing an unlimited number of terms. Such a series is **convergent** if the sum of its terms is a finite quantity. It is **divergent** when the sum of its terms does not approach a finite limit.

Comparison Test. A series is converging if each term in it is equal to or less than the corresponding term of a known converging series.

Converging series for comparison:

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + \cdots \qquad [r<1]$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}} + \cdots$$

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} + \cdots$$

$$1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots \qquad [p>1]$$

A series is diverging if each term in it is equal to or greater than the corresponding term of a known diverging series.

Diverging series for comparison:

$$a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} + \ldots \quad [r \geqq 1]$$

$$1 + 1 + 1 + 1 + 1 + \ldots$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} + \ldots$$

Ratio Test. If, as the number of terms approaches infinity as its limit, the ratio of the $(n + 1)$th term to the nth term approaches some finite limit (a), the series is convergent if (a) is less than 1, divergent if (a) is greater than 1, and indeterminate by this method if $(a) = 1$.

Oscillating Series. A series whose terms are alternately positive and negative is convergent if each term is numerically less than the preceding term.

Standard Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

$$e^{jx} = e^{\sqrt{-1}\,x} = 1 + jx - \frac{x^2}{2!} - \frac{jx^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{-jx} = e^{-\sqrt{-1}\,x} = 1 - jx - \frac{x^2}{2!} + \frac{jx^3}{3!} + \frac{x^4}{4!} - \cdots$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = \lim_{n \doteq \infty} \left(1 + \frac{1}{n}\right)^n$$

$$= 2.7182818 \ldots$$

$$a^x = 1 + x \log a + \frac{(x \log a)^2}{2!} + \frac{(x \log a)^3}{3!} + \cdots$$

$$\log x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right] \quad [x > 0]$$

$$\log x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots \qquad [x > \tfrac{1}{2}]$$

$$\log x = (x-1) - \tfrac{1}{2}(x-1)^2 + \tfrac{1}{3}(x-3)^3 - \ldots \qquad [2 > x > 0]$$

$$(1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + x^4 \mp x^5 + \ldots \qquad [x^2 < 1]$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \qquad [1 \geqq x > -1]$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots \qquad [1 > x \geqq -1]$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \cdots$$

$$\left[\frac{\pi}{2} > x > -\frac{\pi}{2}\right]$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \cdots \qquad [x^2 < \pi^2]$$

$$\sin^{-1} x = x + \frac{x^3}{6} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{x^5}{5} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{x^7}{7} + \cdots \qquad [1 > x > -1]$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad [1 > x > -1]$$

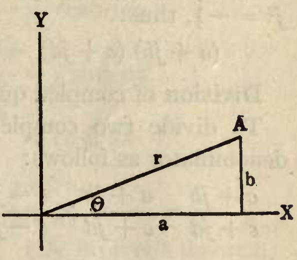
$$\tan^{-1} x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots \qquad [x^2 > 1]$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \cdots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots$$

Complex Imaginary Quantities

The imaginary unit $= \sqrt{-1} = j$



In representing complex imaginary quantities, it is usual to represent real quantities in the direction of the horizontal or X-axis, and imaginaries in the direction of the vertical or Y-axis. Multiplication by the imaginary unit, j, revolves a quantity through 90 degrees, in counter-clockwise direction.

A **complex number** is the sum of a real and an imaginary, thus:

$$A = a + jb = a + \sqrt{-1}\, b$$

is a **complex number.**

A **complex number** may be written in any of the following identical forms:

$$A = a + jb = r(\cos\theta + j\sin\theta) = re^{j\theta} \quad [\theta \text{ in radians}]$$

in which $\begin{cases} a = r\cos\theta, \\ b = r\sin\theta. \end{cases}$

The **magnitude** of the complex number, $a + jb$, is

$$r = \sqrt{a^2 + b^2}$$

Addition and Subtraction of complex quantities:

To add two complex quantities, combine the real parts, and then the imaginaries, thus:

$$(a + jb) + (c + jd) = (a + c) + j(b + d)$$

In the same way, to subtract two complex quantities:

$$(a + jb) - (c + jd) = (a - c) + j(b - d)$$

Multiplication of complex quantities:

To find the product of two complex numbers, multiply out as in ordinary algebra, remembering that $j^2 = -1$, thus:

$$(a + jb)(c + jd) = (ac - bd) + j(ad + bc)$$

Division of complex quantities:

To divide two complex quantities, rationalize the denominator as follows:

$$\frac{a+jb}{c+jd} = \frac{a+jb}{c+jd} \times \frac{c-jd}{c-jd} = \frac{(ac+bd)+j(bc-ad)}{c^2+d^2}$$

Logarithms of complex quantities:

To obtain the logarithm of a complex quantity, use the following formulæ:

$$\log_e(a+jb) = \log_e r(\cos\theta + j\sin\theta), \quad \text{where } r = \sqrt{a^2+b^2}$$

$$= \log_e (r\, e^{j\theta})$$

$$= \log_e r + \log_e e^{j\theta}$$

$$= \log_e r + j\theta$$

$$\log_n(a+jb) = \log_n (r\, e^{j\theta}) = \log_n r + j\theta \log_n e$$

Complex Imaginary Formulæ

$$j = \sqrt{-1}$$

$$j^2 = jj = -1$$

$$e = 2.71828 +$$

$$e^{jax} = \cos ax + j\sin ax = \cosh jax + \sinh jax$$

$$e^{-jax} = \cos ax - j\sin ax = \cosh jax - \sinh jax$$

$$e^{ax} = \cos jax - j\sin jax = \cosh ax + \sinh ax$$

$$e^{-ax} = \cos jax + j\sin jax = \cosh ax - \sinh ax$$

$$\sin ax = \frac{e^{jax} - e^{-jax}}{2j} = \frac{\sinh jax}{j}$$

$$\cos ax = \frac{e^{jax} + e^{-jax}}{2} = \cosh jax$$

$$\sin jax = j\frac{e^{ax} - e^{-ax}}{2} = j \sinh ax$$

$$\cos jax = \frac{e^{ax} + e^{-ax}}{2} = \cosh ax$$

$$e^{u \pm jv} = e^{u}(\cos v \pm j \sin v)$$

$$(\cos \theta + j \sin \theta)^{n} = \cos n\theta + j \sin n\theta$$

(De Moivre's theorem)

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j \sin\frac{\pi}{2} = j$$

$$e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j \sin\frac{\pi}{2} = -j$$

$$e^{j\left(\theta + \frac{\pi}{2}\right)} = e^{j\theta}e^{j\frac{\pi}{2}} = je^{j\theta}$$

$$e^{j\left(\theta - \frac{\pi}{2}\right)} = e^{j\theta}e^{-j\frac{\pi}{2}} = -j\, e^{j\theta}$$

Permutations and Combinations

The number of **permutations** of n different things taken r at a time is

$$P_r = n(n-1)\ .\ .\ .\ (n-r+1) = \frac{n!}{(n-r)!}$$

For n different things taken all at a time, the number of **permutations** is

$$P_n = n(n-1)\ .\ .\ .\ (2)\ (1) = n!$$

The number of **permutations** of n things taken all at a time, n_1 being alike, n_2 alike, n_3 alike, etc., is

$$P = \frac{n!}{n_1!\ n_2!\ n_3!}\ \cdots$$

The number of **combinations** of n things taken r at a time is

$$C_r = \frac{n(n-1)\ldots(n-r+1)}{r!} = \frac{n!}{r!\,(n-r)!}$$

For n things taken 1, 2, 3, . . . n at a time, the total number of **combinations** is

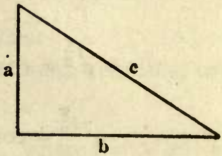
$$C = 2^n - 1$$

GEOMETRY

Plane Figures

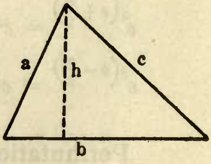
Right Triangle

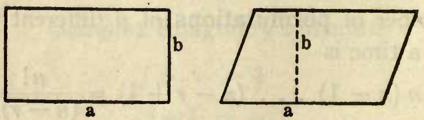
$$c = \sqrt{a^2 + b^2}$$
$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$
$$\text{area} = \tfrac{1}{2}ab$$



Any Triangle

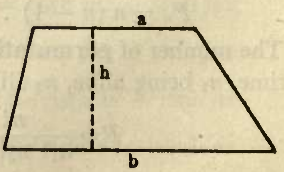
$$\text{area} = \tfrac{1}{2}bh$$
$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$s = \tfrac{1}{2}(a+b+c)$$





Parallelogram

$$\text{area} = ab$$



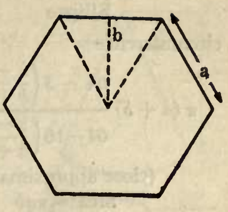
Trapezoid

$$\text{area} = \tfrac{1}{2}h(a+b)$$

Regular Polygon

area $= \frac{1}{2}\,abn$

n = number of sides



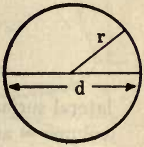
Circle

circumference $= 2\,\pi r$

$= \pi d$

area $= \pi r^2$

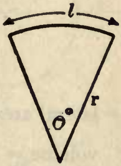
$= \pi \dfrac{d^2}{4}$



Sector of Circle

arc $= l = \pi r \dfrac{\theta^\circ}{180^\circ}$

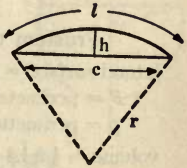
area $= \frac{1}{2}\,rl = \pi r^2 \dfrac{\theta^\circ}{360^\circ}$



Segment of Circle

chord $= c = 2\sqrt{2\,hr - h^2}$

area $= \frac{1}{2}\,rl - \frac{1}{2}\,c\,(r - h)$



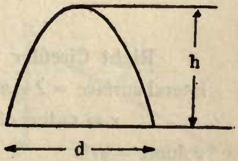
Parabola

length of arc $\simeq \dfrac{d^2}{8\,h}\left[\sqrt{c\,(1+c)} + 2.0326 \log_{10}\left(\sqrt{c} + \sqrt{1+c}\right)\right]$

in which

$c = \left(\dfrac{4\,h}{d}\right)^2$

area $= \frac{2}{3}\,dh$



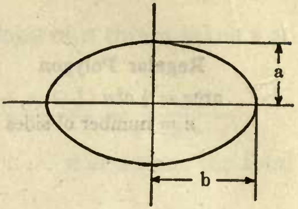
Ellipse

circumference =

$$\pi (a + b) \frac{64 - 3\left(\dfrac{b-a}{b+a}\right)^4}{64 - 16\left(\dfrac{b-a}{b+a}\right)^2}$$

(close approximation)

area $= \pi ab$

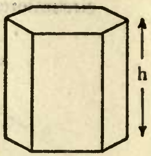


Solids

Right Prism

lateral surface = perimeter of base $\times h$

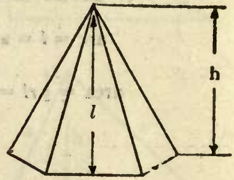
volume = area of base $\times h$



Pyramid

lateral area = $\frac{1}{2}$ perimeter of base $\times l$

volume = area of base $\times \frac{h}{3}$



Frustum of Pyramid

lateral surface $= \frac{1}{2} l (P + p)$

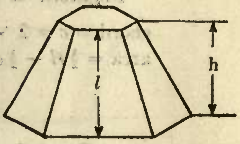
P = perimeter of lower base

p = perimeter of upper base

volume $= \frac{1}{3} h [A + a + \sqrt{Aa}]$

A = area of lower base

a = area of upper base

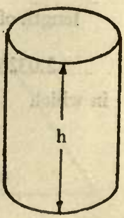


Right Circular Cylinder

lateral surface $= 2\pi rh$

r = radius of base

volume $= \pi r^2 h$

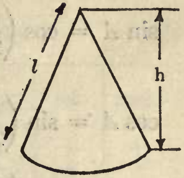


Right Circular Cone

lateral surface $= \pi r l$

r = radius of base

volume $= \frac{1}{3}\pi r^2 h$



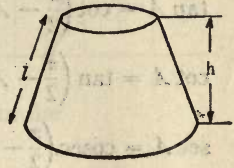
Frustum of Right Circular Cone

lateral surface $= \pi l\,(R + r)$

R = radius of lower base

r = radius of upper base

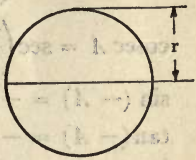
volume $= \frac{1}{3}\pi h\,[R^2 + Rr + r^2]$



Sphere

surface $= 4\pi r^2$

volume $= \frac{4}{3}\pi r^3$



Segment of Sphere

volume of segment

$= \frac{1}{6} a\pi\,[3\,(r_1^2 + r_2^2) + a^2]$

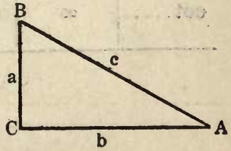
PLANE TRIGONOMETRY

Right Triangle

$\sin A = \frac{a}{c}$ $\quad \cos A = \frac{b}{c}$

$\tan A = \frac{a}{b}$ $\quad \cot A = \frac{b}{a}$

$\sec A = \frac{c}{b}$ $\quad \operatorname{cosec} A = \frac{c}{a}$



$$\sin A = \cos\left(\frac{\pi}{2} - A\right) = -\cos\left(\frac{\pi}{2} + A\right)$$

$$\cos A = \sin\left(\frac{\pi}{2} - A\right) = \sin\left(\frac{\pi}{2} + A\right)$$

$$\tan A = \cot\left(\frac{\pi}{2} - A\right) = -\cot\left(\frac{\pi}{2} + A\right)$$

$$\cot A = \tan\left(\frac{\pi}{2} - A\right) = -\tan\left(\frac{\pi}{2} + A\right)$$

$$\sec A = \operatorname{cosec}\left(\frac{\pi}{2} - A\right) = \operatorname{cosec}\left(\frac{\pi}{2} + A\right)$$

$$\operatorname{cosec} A = \sec\left(\frac{\pi}{2} - A\right) = -\sec\left(\frac{\pi}{2} + A\right)$$

$\sin(-A) = -\sin A$ $\qquad$ $\cos(-A) = \cos A$

$\tan(-A) = -\tan A$ $\qquad$ $\cot(-A) = -\cot A$

$\sec(-A) = \sec A$ $\qquad$ $\operatorname{cosec}(-A) = -\operatorname{cosec} A$

NUMERICAL VALUES

Angle ..	0°	30°	45°	60°	90°
sin.....	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos.....	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan.....	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
cot.....	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

SIGNS OF THE FUNCTIONS

	sin	cos	tan	cot	sec	cosec
1st Quadrant......	+	+	+	+	+	+
2nd Quadrant......	+	−	−	−	−	+
3rd Quadrant......	−	−	+	+	−	−
4th Quadrant......	−	+	−	−	+	−

Trigonometric Formulæ

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\operatorname{cosec}^2 x = 1 + \cot^2 x$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$$

$$\sin \tfrac{1}{2} x = \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \tfrac{1}{2} x = \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \tfrac{1}{2} x = \frac{1 - \cos x}{\sin x}$$

$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$$

$$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$$

$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$$

$$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$$

$$\sin x = \sqrt{1 - \cos^2 x} = \frac{1}{\operatorname{cosec} x} = \frac{\cos x}{\cot x} = \frac{\tan x}{\sec x}$$

$$= \cos x \tan x = \frac{\tan x}{\sqrt{1 + \tan^2 x}} = \frac{1}{\sqrt{1 + \cot^2 x}}$$

$$= \frac{\sqrt{\sec^2 x - 1}}{\sec x} = \frac{\sin 2x}{2 \cos x} = \sqrt{\tfrac{1}{2}(1 - \cos 2x)}$$

$$= 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$\cos x = \sqrt{1 - \sin^2 x} = \frac{1}{\sec x} = \frac{\sin x}{\tan x} = \frac{\cot x}{\operatorname{cosec} x}$$

$$= \sin x \cot x = \frac{\cot x}{\sqrt{1 + \cot^2 x}} = \frac{1}{\sqrt{1 + \tan^2 x}}$$

$$= \sqrt{\frac{\sec^2 x - 1}{\operatorname{cosec} x}} = \frac{\sin 2x}{2 \sin x} = \sqrt{\tfrac{1}{2}(1 + \cos 2x)}$$

$$= \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = 2 \cos^2 \frac{x}{2} - 1 = 1 - 2 \sin^2 \frac{x}{2}$$

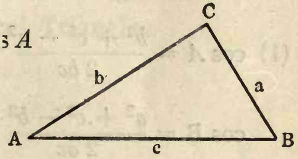
Equivalent expressions for $\tan x$ and $\cot x$ may be obtained by taking any of the above expressions for $\sin x$ and $\cos x$ and substituting in the equations

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Solution of Any Plane Triangle

I. Given any two sides b and c and their included angle A.

Use any one of the following sets of formulas:



(1) $\frac{1}{2}(B + C) = 90° - \frac{1}{2}A$

$$\tan \tfrac{1}{2}(B - C) = \frac{b - c}{b + c} \tan \tfrac{1}{2}(B + C)$$

$$B = \tfrac{1}{2}(B + C) + \tfrac{1}{2}(B - C)$$

$$C = \tfrac{1}{2}(B + C) - \tfrac{1}{2}(B - C)$$

$$a = \frac{b \sin A}{\sin B}$$

(2) $\tan C = \dfrac{c \sin A}{b - c \cos A}$

$$B = 180° - (A + C)$$

$$a = \frac{c \sin A}{\sin C}$$

(3) $a = \sqrt{b^2 + c^2 - 2\,bc \cos A}$

$$\sin B = \frac{b \sin A}{a}$$

$$C = 180° - (A + B)$$

II. Given any two angles A and B and any side c.

$$C = 180° - (A + B)$$

$$a = \frac{c \sin A}{\sin C}$$

$$b = \frac{c \sin B}{\sin C}$$

III. Given the three sides a, b, and c. Use either of the following sets of formulas.

(1) $$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2\,ac}$$

$$C = 180° - (A + B)$$

(2) $$s = \tfrac{1}{2}(a + b + c)$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$\tan \tfrac{1}{2} A = \frac{r}{s-a}$$

$$\tan \tfrac{1}{2} B = \frac{r}{s-b}$$

$$\tan \tfrac{1}{2} C = \frac{r}{s-c}$$

IV. Given any two sides a and b and an angle A opposite either one of these.

$$\sin C = \frac{c \sin A}{a}$$

$$B = 180° - (A + C)$$

$$b = \frac{a \sin B}{\sin A}$$

NOTE. There may be two values for the angle C. If, however, one solution is such that $A + C > 180°$, use other value only.

SPHERICAL TRIGONOMETRY

Right Spherical Triangles

$\cos c = \cos a \cos b$

$\sin a = \sin c \sin A$

$\sin b = \sin c \sin B$

$\cos A = \cos a \sin B$

$\cos B = \cos b \sin A$

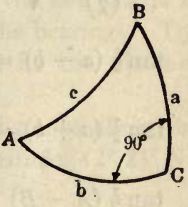
$\cos A = \tan b \cot c$

$\cos B = \tan a \cot c$

$\sin b = \tan a \cot A$

$\sin a = \tan b \cot B$

$\cos c = \cot A \cot B$



Oblique Spherical Triangles

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

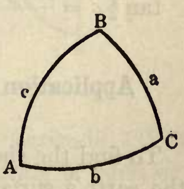
$\cos a = \cos b \cos c + \sin b \sin c \cos A$

$\cos A = \sin B \sin C \cos a - \cos B \cos C$

$\cot a \sin b = \cot A \sin C + \cos C \cos b$

$s = \frac{1}{2}(a + b + c)$

$S = \frac{1}{2}(A + B + C)$



$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}$$

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}}$$

$$\sin\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S \cos(S-A)}{\sin B \sin C}}$$

$$\cos\left(\frac{a}{2}\right) = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B \sin C}}$$

$$\tan\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S \cos(S-A)}{\cos(S-B)\cos(S-C)}}$$

$$\tan\tfrac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c$$

$$\tan\tfrac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c$$

$$\tan\tfrac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C$$

$$\tan\tfrac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C$$

$$\tan\tfrac{1}{2}c = \frac{\sin\frac{1}{2}(A+B)\tan\frac{1}{2}(a-b)}{\sin\frac{1}{2}(A-B)}$$

Application of Spherical Trigonometry to Navigation

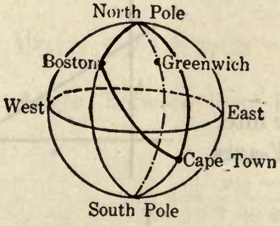
To find the shortest distance between two points on the earth's surface and the bearing of each from the other, the latitude and longitude of each being given. (From W. A. Granville's "Plane and Spherical Trigonometry.")

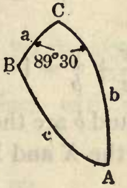
(1) Subtract the latitude of each place algebraically from 90°, taking North latitudes as positive and South latitudes as negative. The results will be the two sides of a spherical triangle.

(2) Find the difference of longitude of the two places by subtracting the lesser longitude from the greater if both are East or both are West; but adding the two if one is East and the other West. This gives the included angle of the triangle. If the difference of longitude found is greater than 180°, then subtract it from 360° and use the remainder as the included angle.

(3) Solving the triangle by the formulæ for $\tan \frac{1}{2}(A - B)$, $\tan \frac{1}{2}(A + B)$, and $\tan \frac{1}{2} c$, the third side gives the shortest distance between the two points in degrees of arc, and the angles give the bearings. The number of minutes in the arc will be the distance between the places in nautical miles.

Illustration. Find the shortest distance along the earth's surface between Boston (latitude 42° 21′ N.,





longitude 71° 4′ W.) and Capetown (latitude 33° 56′ S., longitude 18° 26′ E.) and the bearing of each city from the other.

(1) $a = 90° - 42° \, 21' = 47° \, 39'$

$b = 90° - (-33° \, 56') = 123° \, 56'$

(2) $C = 71° 4' + 18° 26' = 89° 30'$ = difference in longitude.

(3) Solving the triangle as explained above, we get

$c = 68° 14' = 68.23°$ = 4094 nautical miles.

$A = 52° 43'$ = bearing of Boston from Capetown.

$B = 116° 43'$ = bearing of Capetown from Boston.

PLANE ANALYTIC GEOMETRY

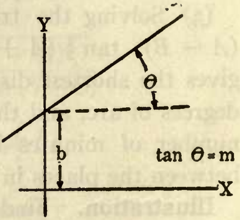
The Straight Line

I. The **slope equation:**

$$y = mx + b$$

m = slope = $\tan\theta$

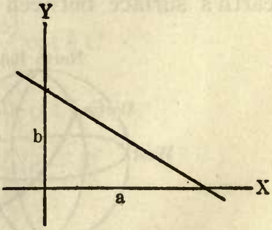
b = intercept on Y-axis



II. The **intercept equation:**

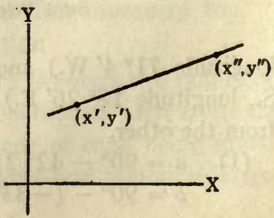
$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the intercepts on the X and Y-axes.



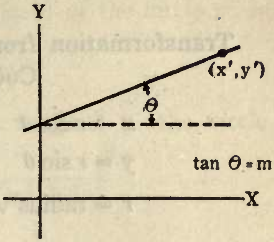
III. Line through the points (x', y') and (x'', y''):

$$\frac{y - y'}{y'' - y'} = \frac{x - x'}{x'' - x'}.$$



IV. Line through the point (x', y'), with slope m:

$$y - y' = m(x - x')$$



V. Distance from the point (x', y') to the line $Ax + By + C = 0$:

$$d = \frac{Ax' + By' + C}{\pm\sqrt{A^2 + B^2}}$$

VI. Distance between the points (x', y') and (x'', y''):

$$d = \sqrt{(x' - x'')^2 + (y' - y'')^2}$$

VII. Area of a triangle with vertices at points (x_1, y_1), (x_2, y_2), and (x_3, y_3).

$$A = \tfrac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

VIII. Angle between two lines with slopes m_1 and m_2.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

NOTE. If $m_1 = m_2$ lines are parallel and if $m_1 = -\dfrac{1}{m_2}$ lines are perpendicular.

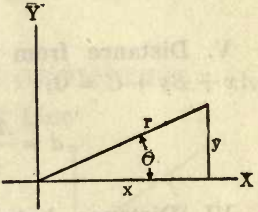
Transformation from Rectangular to Polar Coördinates

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \text{radius vector} = \sqrt{x^2 + y^2}$$

$$\theta = \text{polar angle} = \tan^{-1}\frac{y}{x}$$



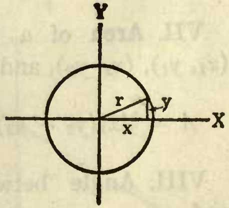
$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$$

The Circle

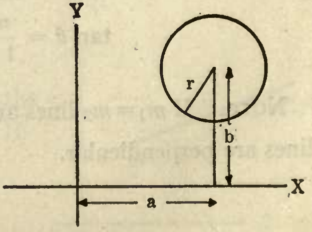
I. Circle of radius r with center at origin:

$$x^2 + y^2 = r^2$$



II. Circle of radius r with its center at the point (a, b):

$$(x - a)^2 + (y - b)^2 = r^2$$



III. Tangent at the point (a, b) of the circle $x^2 + y^2 = r^2$ is

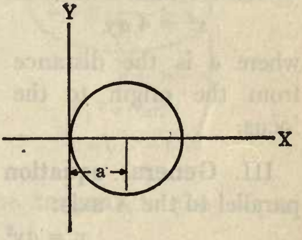
$$ax + by = r^2$$

IV. Slope equation of the **tangent** to the circle $x^2 + y^2 = r^2$ is

$$y = mx \pm r\sqrt{m^2 + 1}$$

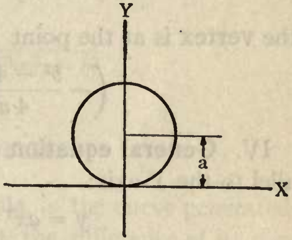
V. Polar equation of circle of radius a passing through the origin, and having its center on the X-axis:

$$r = 2\,a\cos\theta$$



VI. Polar equation of circle of radius a passing through the origin, and having its center on the Y-axis:

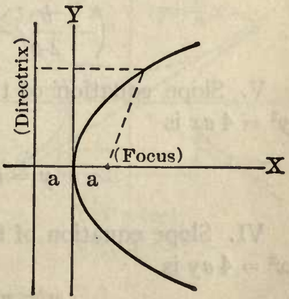
$$r = 2\,a\sin\theta$$



Parabola

Definition. The **parabola** is the curve generated by a point moving so as to remain always equidistant from a given fixed point and a given fixed line.

The fixed point is called the **focus**; the fixed line is called the **directrix.**



I. **Parabola** with its axis along the X-axis and vertex at origin:

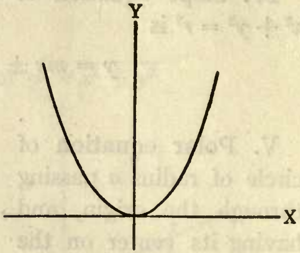
$$y^2 = 4\,ax$$

where a is the distance from the origin to the focus.

II. **Parabola** having its axis along the Y-axis and vertex at origin:

$$x^2 = 4\,ay$$

where a is the distance from the origin to the focus.



III. **General equation** of a parabola with axis parallel to the X-axis:

$$x = ay^2 + by + c$$

the **vertex** is at the point

$$\left(-\frac{b^2 - 4\,ac}{4\,a}, -\frac{b}{2\,a}\right)$$

IV. **General equation** of a parabola with axis parallel to the Y-axis:

$$y = ax^2 + bx + c$$

the **vertex** is at the point

$$\left(-\frac{b}{2\,a}, -\frac{b^2 - 4\,ac}{4\,a}\right)$$

V. Slope equation of the **tangent** to the parabola $y^2 = 4\,ax$ is

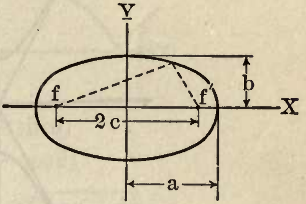
$$y = mx + \frac{a}{m}$$

VI. Slope equation of the **tangent** to the parabola $x^2 = 4\,ay$ is

$$y = mx - am^2$$

Ellipse

Definition. The **ellipse** is the curve generated by a point moving so that the sum of its distances from two fixed points is always constant. The fixed points are called the **foci.**



f , f′ = foci

$c = \sqrt{a^2 - b^2}$

I. Equation of ellipse with **center at origin:**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

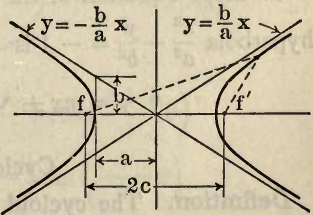
where a and b are one-half the major and minor axes.

II. Slope equation of the **tangent** to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

Hyperbola

Definition. The **hyperbola** is the curve generated by a point moving so that the difference of its distances from two fixed points is always constant.



f , f′ = foci

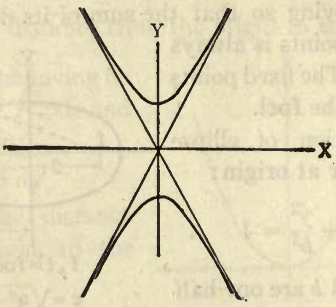
$c = \sqrt{a^2 + b^2}$

I. Equation of hyperbola with **center at origin:**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

II. Equation of **conjugate hyperbola:**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$



III. Equations of **asymptotes** of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are

$$y = \frac{b}{a}x \qquad y = -\frac{b}{a}x$$

IV. Slope equation of the **tangent** to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

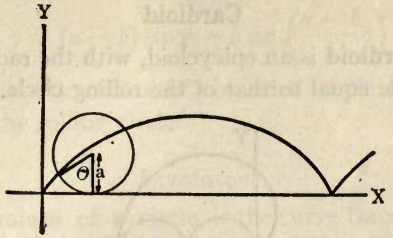
$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

V. Slope equation of the **tangent** to the conjugate hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

$$y = mx \pm \sqrt{b^2 - a^2m^2}$$

Cycloid

Definition. The **cycloid** is the curve generated by a point on the circumference of a circle as the circle rolls along a straight line.

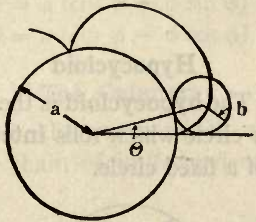


$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$
$$x = a \operatorname{vers}^{-1}\frac{y}{a} - \sqrt{2ay - y^2}$$

where a is the radius of the rolling circle.

Epicycloid

Definition. The **epicycloid** is the curve generated by a fixed point on the circumference of a circle which rolls **externally** on the circumference of a fixed circle.

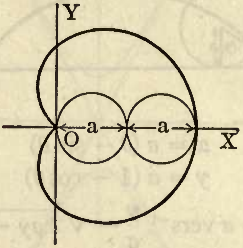


$$x = (a+b)\cos\theta - b\cos\left(\frac{a+b}{b}\theta\right)$$
$$y = (a+b)\sin\theta - b\sin\left(\frac{a+b}{b}\theta\right)$$

where a is the radius of the fixed circle, and b the radius of the rolling circle.

Cardioid

The **cardioid** is an epicycloid, with the radius of the fixed circle equal to that of the rolling circle.



$$r = a\,(1 + \cos\theta)$$

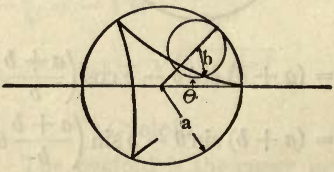
$$x = a\cos\theta\,(1 + \cos\theta)$$

$$y = a\sin\theta\,(1 + \cos\theta)$$

$$\text{Area} = \frac{3\pi a^2}{2} \qquad \text{Length} = 8a$$

Hypocycloid

Definition. The **hypocycloid** is the curve generated by a point on a circle which rolls **internally** along the circumference of a fixed circle.



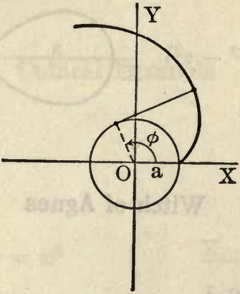
$$x = (a - b)\cos\theta + b\cos\left(\frac{a - b}{b}\,\theta\right)$$

$$y = (a - b)\sin\theta - b\sin\left(\frac{a-b}{b}\theta\right)$$

where a is the radius of the fixed circle and b the radius of the rolling circle.

Involute

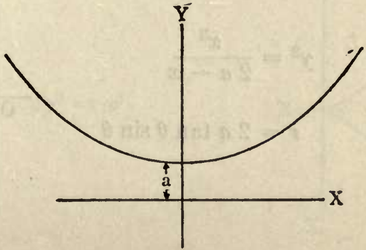
The **involute of a circle** is the curve traced by the end of a taut string which is unwound from the circumference of a fixed circle.



$$x = a(\cos\phi + \phi\sin\phi)$$
$$y = a(\sin\phi - \phi\cos\phi)$$

The Catenary

The **catenary** is the curve which a heavy cord or perfectly flexible chain of uniform density forms, due



to its own weight, when freely suspended between two points.

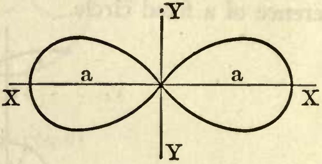
$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) = a \cosh \frac{x}{a}$$

Lemniscate

$r^2 = a^2 \cos 2\theta$

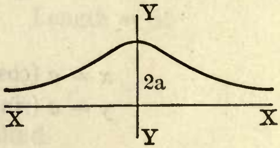
$(x^2 + y^2)^2 = a^2(x^2 - y^2)$

Area $= a^2$



Witch of Agnes

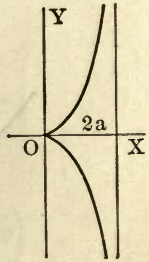
$$y = \frac{8a^3}{x^2 + 4a^2}$$



Cissoid

$$y^2 = \frac{x^3}{2\,a - x}$$

$$r = 2\,a \tan \theta \sin \theta$$

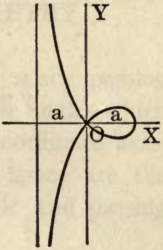


Strophoid

$$y^2 = x^2\left(\frac{a-x}{a+x}\right)$$

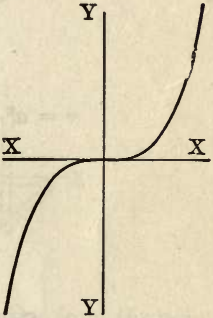
$$r = a\,(\cos\theta - \sin\theta\tan\theta)$$

$$\text{Area of loop} = \frac{a^2}{2}(4-\pi)$$



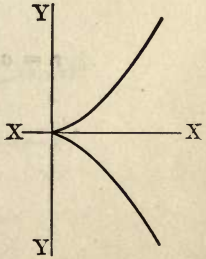
Cubical Parabola

$$a^2y = x^3$$



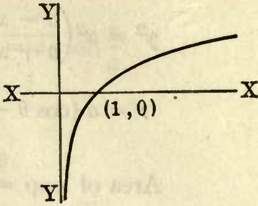
Semi-cubical Parabola

$$ay^2 = x^3$$



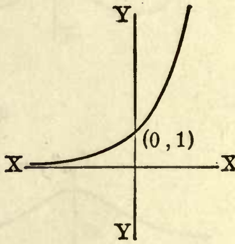
Logarithmic Curve

$y = \log_a x$



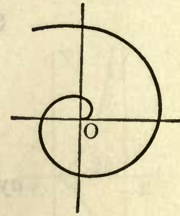
Exponential Curve

$y = a^x$



Spiral of Archimedes

$r = a\,\theta$

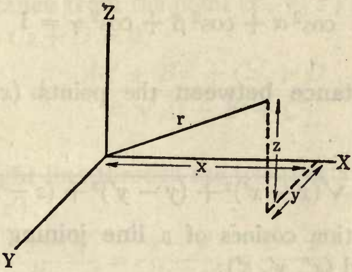


SOLID ANALYTIC GEOMETRY

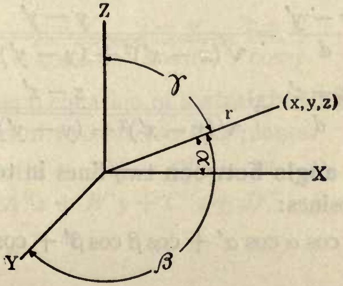
The **direction cosines** of a line in space passing through the origin are the cosines of the angles which the line makes with the rectangular coördinate axes. The direction cosines of **any line** in space are the direction cosines of a line parallel to it and passing through the origin.

I. **Distance from** the point (x, y, z) to the origin:

$$r = \sqrt{x^2 + y^2 + z^2}$$



II. The **direction cosines** of the line from the point (x, y, z) to the origin are:



$$\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\beta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\gamma = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

III. The **sum of the squares** of the direction cosines of a line is equal to 1,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

IV. **Distance between** the points (x, y, z) and (x', y', z'):

$$d = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

V. Direction cosines of a **line joining the points** (x, y, z) and (x', y', z'):

$$\cos\alpha = \frac{x-x'}{d} = \frac{x-x'}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

$$\cos\beta = \frac{y-y'}{d} = \frac{y-y'}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

$$\cos\gamma = \frac{z-z'}{d} = \frac{z-z'}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

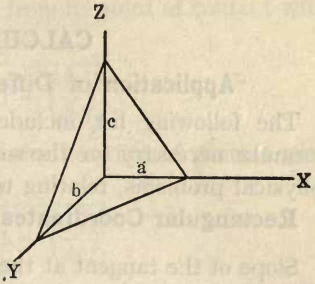
VI. The **angle between** two lines in terms of their direction cosines:

$$\cos\theta = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$$

VII. **Intercept equation** of a plane:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

where a, b, and c are the intercepts of the plane on the X, Y, and Z axes.



VIII. General equation of a **plane**:

$$Ax+By+Cz+D=0$$

IX. **Distance from** the point (x', y', z') to the plane $Ax+By+Cz+D=0$:

$$d=\frac{Ax'+By'+Cz'+D}{\pm\sqrt{A^2+B^2+C^2}}$$

X. **Straight line** through the two points (x'', y'', z'') and (x', y', z'):

$$\frac{x-x'}{x''-x'}=\frac{y-y'}{y''-y'}=\frac{z-z'}{z''-z'}$$

XI. **Straight line** through the point (x', y', z'), and making the angles α, β, and γ with the coördinate axes:

$$\frac{x-x'}{\cos\alpha}=\frac{y-y'}{\cos\beta}=\frac{z-z'}{\cos\gamma}$$

XII. General equation of a **straight line** is given by the equations of two intersecting planes:

$$A'x+B'y+C'z+D'=0$$
$$A''x+B''y+C''z+D''=0$$

CALCULUS

Application of Differential Calculus

The following list includes some of the principal formulæ necessary for the solution of geometrical and physical problems, relating to any curve $y = f(x)$.

Rectangular Coördinates:

Slope of the tangent at the point $(x, y) = \frac{dy}{dx}$

Slope of the normal $= -\frac{dx}{dy}$

Equation of the tangent at the point (x_o, y_o), x_o and y_o being the coördinates of the given point, is

$$y_o - y = \frac{dy_o}{dx_o}(x_o - x)$$

Equation of the normal at (x_o, y_o) is

$$(y_o - y) = -\frac{dx_o}{dy_o}(x_o - x)$$

The intercept of the tangent on the X-axis is $x - y\frac{dx}{dy}$

The intercept of the tangent on the Y-axis is $y - x\frac{dy}{dx}$

The intercept of the normal on the X-axis is $x + y\frac{dy}{dx}$

The intercept of the normal on the Y-axis is $y + x\frac{dx}{dy}$

Length of the tangent from its point of contact with the curve to the X-axis is

$$y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

Length of the tangent from its point of contact with the curve to the Y-axis is

$$x\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Length of the normal from its point of contact with the curve to the X-axis is

$$y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Length of the normal from its point of contact with the curve to the Y-axis is

$$x\sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

Length of the subtangent $= y\dfrac{dx}{dy}$

Length of the subnormal $= y\dfrac{dy}{dx}$

Differential length of the arc $= ds = \sqrt{(dx)^2 + (dy)^2}$

$$= dy\sqrt{1+\left(\frac{dx}{dy}\right)^2} = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Radius of curvature $= \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$

Curvature is the reciprocal of radius of curvature.

Length of the perpendicular from the origin on the tangent (to the curve) is

$$\frac{x\dfrac{dy}{dx} - y}{\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}}$$

Polar Coördinates:

$\tan\psi = r\,\dfrac{d\theta}{dr}$, where ψ is the angle between the radius vector and that part of the tangent to the curve at (r, θ) drawn back toward the initial line.

Length of polar subtangent $= r^2 \dfrac{d\theta}{dr}$

Length of polar subnormal $= \dfrac{dr}{d\theta}$

Differential length of arc $= ds = \sqrt{(dr)^2 + r^2\,(d\theta)^2}$

$$= dr\sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2} = d\theta\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

Length of the perpendicular from the pole on the tangent $= p = r^2\dfrac{d\theta}{ds}$, also,

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

Formulæ of Differential Calculus

$$d\,(au) = a\,du$$
$$d\,(u+v) = du + dv$$
$$d\,(uv) = v\,du + u\,dv$$
$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$
$$d\,(x^n) = nx^{n-1}\,dx$$
$$d\,(x^y) = yx^{y-1}\,dx + x^y \log_e x\,dy$$
$$d\,(e^x) = e^x\,dx$$
$$d\,(a^u) = a^u \log_e a\,du$$
$$d\,(\log_e x) = \frac{1}{x}\,dx$$

$$d(\sin x) = \cos x\, dx$$
$$d(\cos x) = -\sin x\, dx$$
$$d(\tan x) = \sec^2 x\, dx$$
$$d(\cot x) = -\operatorname{cosec}^2 x\, dx$$
$$d(\sec x) = \sec x \tan x\, dx$$
$$d(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x\, dx$$
$$d(\sin^{-1} x) = \frac{dx}{\sqrt{1-x^2}}$$
$$d(\cos^{-1} x) = -\frac{dx}{\sqrt{1-x^2}}$$
$$d(\tan^{-1} x) = \frac{dx}{1+x^2}$$
$$d(\cot^{-1} x) = -\frac{dx}{1+x^2}$$
$$d(\sec^{-1} x) = \frac{dx}{x\sqrt{x^2-1}}$$
$$d(\operatorname{cosec}^{-1} x) = -\frac{dx}{x\sqrt{x^2-1}}$$

Maxima and Minima

The **maximum** or **minimum** values of a given function $y = f(x)$ are obtained as follows:

(1) Find the first derivative $\frac{dy}{dx}$ and equate it to zero.

(2) Solve the resulting equation for values of x.

(3) In order to determine whether these values of x make y maximum or minimum, obtain the second derivative $\frac{d^2y}{dx^2}$ of the given function.

(4) Substitute separately in the expression for $\frac{d^2y}{dx^2}$

each of the values of x found above. Values of x that make $\frac{d^2y}{dx^2}$ positive correspond to minimum values of the function, and values of x that make $\frac{d^2y}{dx^2}$ negative correspond to maximum values of the function.

(5) Substituting these values of x in the given function $y = f(x)$, we obtain the maximum or minimum values of y.

Illustrative Example. Find the values of x which will make the function $y = 6x + 3x^2 - 4x^3$ a maximum or a minimum, and find the corresponding values of the function y.

(1) The first derivative of y is

$$\frac{dy}{dx} = 6 + 6x - 12x^2$$

(2) The values of x which make y maximum or minimum will make $\frac{dy}{dx} = 0$; therefore

$$6 + 6x - 12x^2 = 0, \quad \text{or} \quad x^2 - \tfrac{1}{2}x = \tfrac{1}{2}$$

solving, $\quad x = \frac{1}{4} \pm \frac{3}{4} = +1 \quad \text{or} \quad -\frac{1}{2}$

Hence, the maximum or minimum values of y must occur when $x = 1$ or $-\frac{1}{2}$.

(3) To determine whether these values are maxima or minima, we obtain the second derivative of y; thus:

$$\frac{d^2y}{dx^2} = 6 - 24x$$

(4) When $x = 1$, $\frac{d^2y}{dx^2} = -18$, which corresponds to a maximum value of y.

When $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} = +18$, which corresponds to a minimum value of y.

(5) Substituting these values of x in the given function, we have

when $x = 1$, $y = 6 + 3 - 4 = 5$, a maximum

when $x = -\frac{1}{2}$, $y = -3 + \frac{3}{4} + \frac{1}{2} = -\frac{7}{4}$, a minimum

Taylor's and Maclaurin's Series

Taylor's Series:

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \ldots$$

or

$$f(a+x) = f(a) + \frac{x}{1!}f'(a) + \frac{x^2}{2!}f''(a) + \frac{x^3}{3!}f'''(a) + \ldots$$

where $f(a)$ denotes the value of the function when a is substituted for x, $f'(a)$ the value of the first derivative when a is substituted for x, $f''(a)$ the value of the second derivative when a is substituted for x, etc.

Illustrative Examples. Expand $\cos(a + x)$ in powers of x. Here

$$f(a + x) = \cos(a + x)$$

Placing $x = 0$, $\quad f(a) = \cos a$

$f'(a + x) = -\sin(a + x)$, $\quad f'(a) = -\sin a$

$f''(a + x) = -\cos(a + x)$, $\quad f''(a) = -\cos a$

$f'''(a + x) = \sin(a + x)$, $\quad f'''(a) = \sin a$

Substituting in Taylor's formula,

$$\cos(a+x) = \cos a - \frac{x}{1!}\sin a - \frac{x^2}{2!}\cos a + \frac{x^3}{3!}\sin a + \ldots$$

Maclaurin's Series.

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$

where $f(0)$ denotes the value of the function when 0 is substituted for x, $f'(0)$ the value of the first derivative when 0 is substituted for x, etc.

Illustrative Example. Expand $\cos x$ in powers of x.

Here $\qquad f(x) = \cos x$

$$\begin{array}{llll} f(0) = \cos 0 = 1 & & & \\ f'(x) = -\sin x, & & f'(0) = 0 \\ f''(x) = -\cos x, & & f''(0) = -1 \\ f'''(x) = \sin x, & & f'''(0) = 0 \\ f^{\text{iv}}(x) = \cos x, & & f^{\text{iv}}(x) = 1 \end{array}$$

Substituting in Maclaurin's formula,

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

APPLICATION OF INTEGRAL CALCULUS

Lengths of Curves

Rectangular Coördinates:

$$\text{length of curve} = s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

From the equation of the given curve, find y in terms

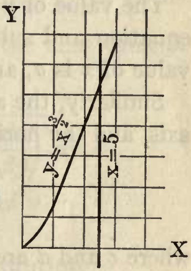
of x; then differentiate in order to obtain $\frac{dy}{dx}$, and substitute its value in the formula. The lower limit a is the initial value of x, and the upper limit b the final value of x.

Or, similarly, by solving for x in terms of y, and obtaining $\frac{dx}{dy}$, the length of the curve is given by the formula

$$s = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

where c and d are the initial and final values of y.

Illustrative Example. Find the length of the arc of the semi-cubical parabola $y^2 = x^3$ from the origin to the ordinate $x = 5$.



$$y = x^{3/2}, \qquad \frac{dy}{dx} = \frac{3}{2}x^{1/2},$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{9x}{4}$$

The required length of arc is

$$S = \int_0^5 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^5 \sqrt{1 + \frac{9x}{4}}\, dx$$

$$= \frac{8}{27}\sqrt{\left(1 + \frac{9x}{4}\right)^3}\,\Bigg]_0^5 = \frac{335}{27}$$

Polar Coördinates:

$$\text{length of curve} = s = \int_a^b \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$$

where a and b are the limiting values of r.

Or,

$$\text{length of curve} = s = \int_{\theta'}^{\theta''} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

where θ' and θ'' are the limiting values of θ.

Plane Areas

Rectangular Coördinates:

The area included between a curve, the X-axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{area} = A = \int_a^b y\, dx$$

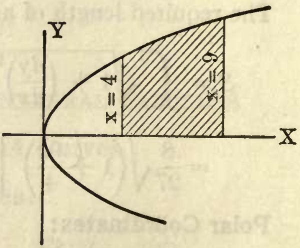
The value of y in terms of x is found from the given equation and substituted in the formula. The initial value of x is a, and the final value b.

Similarly, the area included between a curve, the Y-axis, and the horizontal lines $y = c$ and $y = d$ is

$$\text{area} = A = \int_c^d x\, dy$$

where c and d are the limits of y.

Illustrative Example. Find the area bounded by the parabola $y^2 = 4x$, the axis of X, and the lines $x = 4$, $x = 9$.



$$y^2 = 4x, \qquad y = 2\sqrt{x}$$

The required area is

$$A = \int_4^9 y\, dx = \int_4^9 2\sqrt{x}\, dx$$

$$= \frac{4}{3} x^{3/2}\Big]_4^9 = 25\tfrac{1}{3}$$

Polar Coördinates:

The area included between a given curve and two given radii is

$$\text{area} = A = \frac{1}{2}\int_{\theta'}^{\theta''} r^2\, d\theta$$

where θ'' and θ' are the limiting values of θ.

Areas of Surfaces of Revolution

For revolution about the X-axis,

$$\text{area} = A = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

where the value of $\left(\frac{dy}{dx}\right)$ is found from the given equation. The initial value of x is a, and the final value b.

For revolution about the Y-axis,

$$\text{area} = A = 2\pi \int_c^d x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

where c and d are the limiting values of y.

Volumes of Solids of Revolution

Rectangular Coördinates:

$$\text{volume} = V_x = \pi \int_a^b y^2\, dx$$

is the formula for the volume generated by revolving the given curve about the X-axis. The limiting values of x are a and b.

Similarly, the volume generated by revolving the plane figure about the Y-axis equals

$$V_y = \pi \int_c^d x^2\, dy$$

where c and d are the initial and final values of y.

Polar Coördinates:

When the plane figure is revolved about the X-axis, the volume generated is

$$V_x = 2\pi \int\int r^2 \sin\theta \, d\theta \, dr$$

For revolution about the Y-axis, the volume generated is

$$V_y = 2\pi \int\int r^2 \cos\theta \, d\theta \, dr$$

INDETERMINATE FORMS

If the fraction $\dfrac{f(x)}{F(x)}$ gives rise to the indeterminate form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$, when x approaches a as a limit, the indeterminate form may be replaced by a new fraction, $\dfrac{f'(x)}{F'(x)}$, the numerator of which is equal to the derivative of the given numerator, and the new denominator is equal to the derivative of the given denominator. The value of this new fraction, as x approaches a, is the limiting value of the given fraction. If this again becomes indeterminate, it may be necessary to repeat the process several times.

Example. Find the limiting value, when $x = 1$, of the fraction

$$\frac{x^2 + x - 2}{x^2 - 1}$$

$$\frac{f(x)}{F(x)} = \frac{x^2 + x - 2}{x^2 - 1} = \frac{0}{0}, \text{ when } x = 1$$

$$\frac{f'(x)}{F'(x)} = \frac{2x + 1}{2x} = \frac{3}{2}, \text{ when } x = 1$$

Hence, the required limiting value is $\frac{3}{2}$.

SOLUTION OF EQUATIONS

Algebraic equations may be solved by Newton's method of approximation. Thus, let it be required to solve an equation of the form $Ax^3 + Bx^2 + Cx = D$. Find, by trial, a number, r, nearly equal to the root sought, and let $r + h$ denote the exact value of the root, where h is a small quantity the value of which must be determined. Substituting $r + h$ for x in the given equation and neglecting all powers of h higher than the first, we have, approximately,

$$h = \frac{Ar^3 + Br^2 + Cr - D}{-3\,Ar^2 - 2\,Br - C}$$

It will be observed that the numerator of the above fraction is the first member of the given equation after D has been transposed and x changed to r, and the denominator is the **first derivative** of the numerator with its sign reversed. The correction h added, with its proper sign, to the assumed root r, gives a closer approximation to the value of x. Repeat the operation with the corrected value of r, and a second correction will be obtained which will give a nearer value of the root; two corrections generally give sufficient accuracy.

Illustration. Find a root of the equation

$$x^3 + 2\,x^2 + 3\,x = 50$$

The value of h is

$$h = \frac{r^3 + 2\,r^2 + 3\,r - 50}{-3\,r^2 - 4\,r - 3}$$

By trial, we find that x is nearly equal to 3. On substituting 3 for r, we have

$$h = -\frac{2}{21} = -0.1, \text{ approximately}$$

Hence, $x = 2.9$, nearly. If we substitute this new value of r, the new value of h equals $+0.00228$. Hence $x = 2.90228$. If we repeat the operation with this last value of r, the value of h is then found to be $+0.0000034$. Hence $x = 2.9022834$.

CURVE TRACING

The usual method of tracing curves consists in assigning a series of different values to one of the variables, and calculating the corresponding series of values of the other, thus determining a definite number of points on the curve. By drawing a curve through these points, we obtain a graphical representation of the given equation.

The **general form** and **peculiarities** of the curve can be easily determined and sketched by the following steps:

(1) If possible, solve the equation of the given curve for one of its variables, y for example. If the equation then contains only even powers of x, it is symmetrical with the Y-axis.

Or if, when solved for x, it contains only even powers of y, it is symmetrical with the X-axis.

(2) Find the points in which the curve cuts the axes by solving the equation of the given curve in turn with the equations $x = 0$ and $y = 0$.

(3) Find the values of x, if any, which make y infinite; similarly, test for infinite values of x.

(4) Find the value of the first derivative $\frac{dy}{dx}$; and

thence deduce the maximum and minimum points of the curve.

In **tracing polar curves,** write the equation, if possible, in the form $r = f(\theta)$; and give θ such values as make r easily found, as for example, $0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$, etc.

Putting $\frac{dr}{d\theta} = 0$, we find the values of θ for which r is a maximum or minimum.

METHODS OF INTEGRATION

(By **parts, substitution,** etc.)

When the numerator of a fraction contains a variable to an **equal** or a **higher** power than the denominator, the fraction must be reduced to a mixed quantity (by actually dividing the denominator into the numerator) before it can be integrated.

If an expression cannot be integrated by the formulæ given in the table of integrals, one of the following methods may be used to obtain a solution.

Partial Fractions

A fraction may be resolved into partial fractions, which can be integrated separately.

Example. To integrate

$$\frac{1}{(x+a)(x+b)}dx$$

Let

$$\frac{1}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

where we must determine A and B.

Clearing of fractions,

$$1 = A(x+b) + B(x+a) = (A+B)x + (bA+aB)$$

The coefficients of like powers of x on both sides of the equation are equal; therefore,

$$A + B = 0$$
$$bA + aB = 1$$

whence $\quad A = \dfrac{1}{b-a} \quad$ and $\quad B = \dfrac{1}{a-b}$

and

$$\int \frac{1}{(x+a)(x+b)}\,dx = \int \frac{\left(\dfrac{1}{b-a}\right)}{(x+a)}\,dx + \int \frac{\left(\dfrac{1}{a-b}\right)}{(x+b)}\,dx$$

These forms are now integrable by the table of integrals, the result being

$$\int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\log(x+a) + \frac{1}{a-b}\log(x+b) + C$$

where C is the constant of integration.

Integration by Parts

To integrate by parts, apply the formula

$$\int u\,dv = uv - \int v\,du$$

The method of integration by parts is most effective in dealing with the integration of **products,** involving logarithms, and trigonometric and inverse circular functions.

Generally, the most complicated quantity which can be integrated directly by one of the fundamental formulæ (see Table of Integrals, page 57) is equated, with the differential, to dv, and the remaining part is equated to u.

Example. To find

$$\int x \log(x)\,dx$$

Let $\quad u = \log x \quad$ and $\quad dv = x\,dx$

then $\quad du = \dfrac{dx}{x} \qquad v = \displaystyle\int x\,dx = \frac{x^2}{2}$

Substituting in the formula

$$\int u\,dv = uv - \int v\,du$$

we have

$$\int x \log (x)\,dx = \log (x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\frac{dx}{x}$$

$$= \frac{x^2}{2}\log (x) - \frac{x^2}{4} + C$$

Integration by Substitution

I. Differentials containing fractional powers of x may be integrated by the substitution

$$x = z^n$$

where n is the least common denominator of the fractional exponents of x.

II. Expressions involving only fractional powers of $(a + bx)$ may be rationalized by the substitution

$$(a + bx) = z^n$$

where n is the least common denominator of the fractional exponents of $(a + bx)$.

III. To integrate expressions containing

$$\sqrt{x^2 + ax + b},$$

use the substitution

$$\sqrt{x^2 + ax + b} = z - x$$

IV. Expressions containing $\sqrt{-x^2 + ax + b}$ may be rationalized by the substitution

$$\sqrt{-x^2 + ax + b} = (x - \theta)\,z$$

where $(x - \theta)$ is a factor of $(-x^2 + ax + b)$.

V. A differential containing $\sin x$ and $\cos x$ can be transformed by means of the substitution

$$\tan\frac{x}{2} = z$$

from which

$$\sin x = \frac{2z}{1+z^2} \qquad \cos x = \frac{1-z^2}{1+z^2} \qquad dx = \frac{2\,dz}{1+z^2}$$

VI. A very useful substitution is

$$x = \frac{1}{z}$$

VII. Differentials involving $\sqrt{a^2 - x^2}$ may be rationalized by the substitution

$$x = a\sin\theta$$

VIII. Differentials involving $\sqrt{a^2 + x^2}$ may be rationalized by the substitution

$$x = a\tan\theta$$

IX. Differentials involving $\sqrt{x^2 - a^2}$ may be rationalized by the substitution

$$x = a\sec\theta$$

Reduction Formulæ

The purpose of the following reduction formulæ is to simplify an integral of the form

$$\int x^m (a + bx^n)^p\,dx$$

$$\int x^m (a + bx^n)^p\,dx = \frac{x^{m-n+1}(a+bx^n)^{p+1}}{(np+m+1)\,b} - \frac{(m-n+1)\,a}{(np+m+1)\,b}\int x^{m-n}(a+bx^n)^p\,dx$$

This formula enables us to lower the exponent of x by n, without affecting the exponent of $(a + bx^n)$.

Method fails when $(np + m + 1) = 0$.

II. $$\int x^m (a + bx^n)^p \, dx = \frac{x^{m+1}(a + bx^n)^p}{(np + m + 1)} + \frac{npa}{(np + m + 1)} \int x^m (a + bx^n)^{p-1} \, dx$$

By this formula, the exponent of $(a + bx^n)$ is lowered by 1, without affecting the exponent of x.

Method fails when $(np + m + 1) = 0$.

III. $$\int x^m (a + bx^n)^p \, dx = \frac{x^{m+1}(a + bx^n)^{p+1}}{(m + 1)a} - \frac{(np + m + 1 + n)\, b}{(m + 1)\, a} \int x^{m+n} (a + bx^n)^p \, dx$$

By this formula, the exponent of x is increased by n, without affecting the exponent of $(a + bx^n)$.

Method fails when $m = -1$.

IV. $$\int x^m (a + bx^n)^p \, dx = -\frac{x^{m+1}(a + bx^n)^{p+1}}{n\,(p + 1)\, a} + \frac{(np + n + m + 1)}{n\,(p + 1)\, a} \int x^m (a + bx^n)^{p+1} \, dx$$

This formula enables us to increase the exponent of $(a + bx^n)$ by 1, without affecting the exponent of x.

Method fails when $p = -1$.

TABLE OF INTEGRALS

Fundamental Forms

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int \frac{dx}{x} = \log x$$

$$\int e^x\,dx = e^x$$

$$\int a^x\,dx = \frac{a^x}{\log_e a}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x$$

$$\int \sin x\,dx = -\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int \tan x\,dx = \log(\sec x)$$

$$\int \cot x\,dx = \log(\sin x)$$

$$\int \sec x\,dx = \log\left[\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right]$$

$$\int \operatorname{cosec} x\,dx = \log\left(\tan\frac{x}{2}\right)$$

$$\int \tan x \sec x\,dx = \sec x$$

$$\int \cot x \operatorname{cosec} x\,dx = -\operatorname{cosec} x$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x$$

Expressions involving $(a+bx)$:

$$\int \frac{dx}{(a+bx)} = \frac{1}{b}\log(a+bx)$$

$$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{x\,dx}{(a+bx)} = \frac{1}{b^2}[a + bx - a\log(a+bx)]$$

$$\int \frac{x\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[\log(a+bx) + \frac{a}{a+bx}\right]$$

$$\int \frac{x^2dx}{a+bx} = \frac{1}{b^3}[\tfrac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2\log(a+bx)]$$

$$\int \frac{x^2dx}{(a+bx)^2} = \frac{1}{b^3}\left[(a+bx) - 2a\log(a+bx) - \frac{a^2}{a+bx}\right]$$

$$\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}$$

$$\int \frac{x\,dx}{(a+bx)^3} = \frac{1}{b^2}\left[-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2}\right]$$

$$\int \frac{x^2\,dx}{(a+bx)^3} = \frac{1}{b^3}\left[\log(a+bx) + \frac{2a}{a+bx} - \frac{a^2}{2(a+bx)^2}\right]$$

$$\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\log\frac{a+bx}{x}$$

$$\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2}\log\frac{a+bx}{x}$$

$$\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2}\log\frac{a+bx}{x}$$

$$\int \frac{dx}{x^2(a+bx)^2} = -\frac{a+2bx}{a^2x(a+bx)} + \frac{2b}{a^3}\log\frac{a+bx}{x}$$

$$\int (a+bx)^n dx = \frac{1}{b(n+1)}(a+bx)^{n+1}$$

$$\int x(a+bx)^n\,dx = \frac{1}{b^2(n+2)}(a+bx)^{n+2} - \frac{a}{b^2(n+1)}(a+bx)^{n+1}$$

$$\int x^2 (a + bx)^n \, dx =$$

$$\frac{1}{b^3}\left[\frac{(a+bx)^{n+3}}{n+3} - 2a\frac{(a+bx)^{n+2}}{n+2} + a^2\frac{(a+bx)^{n+1}}{n+1}\right]$$

$$\int \frac{dx}{(a + bx)(c + dx)} = \frac{1}{ad - bc} \log \frac{c + dx}{a + bx}$$

$$\int \frac{dx}{(a + bx)^2 (c + dx)}$$

$$= \frac{1}{ad - bc}\left[\frac{1}{a + bx} + \frac{d}{ad - bc} \log \frac{c + dx}{a + bx}\right]$$

Expressions involving $(a + bx^2)$ or $(a^2 \pm x^2)$:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a + x}{a - x}$$

$$\int \frac{dx}{a + bx^2} = \frac{1}{\sqrt{ab}} \tan^{-1}\left(x\sqrt{\frac{b}{a}}\right) \quad \text{or}$$

$$\int \frac{dx}{a + bx^2} = \frac{1}{2\sqrt{-ab}} \log \frac{\sqrt{a} + x\sqrt{-b}}{\sqrt{a} - x\sqrt{-b}} \quad \text{if } a > 0, b < 0$$

$$\int \frac{dx}{(a + bx^2)^2} = \frac{x}{2a(a + bx^2)} + \frac{1}{2a}\int \frac{dx}{a + bx^2}$$

$$\int \frac{x\,dx}{a + bx^2} = \frac{1}{2b} \log \left(x^2 + \frac{a}{b}\right)$$

$$\int \frac{x^2\,dx}{a + bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a + bx^2}$$

$$\int \frac{dx}{x(a + bx^2)} = \frac{1}{2a} \log \frac{x^2}{a + bx^2}$$

$$\int \frac{dx}{(a + bx^2)^n} = \frac{1}{2(n-1)a} \frac{x}{(a + bx^2)^{n-1}}$$

$$+ \frac{2n - 3}{2(n-1)a}\int \frac{dx}{(a + bx^2)^{n-1}} \quad (n \text{ integer} > 1)$$

$$\int (a + bx^2)^n \, x dx = \frac{1}{2b} \frac{(a + bx^2)^{n+1}}{n+1}$$

$$\int \frac{x^2 \, dx}{(a + bx^2)^n} = -\frac{1}{2(n-1)b} \frac{x}{(a + bx^2)^{n-1}} + \frac{1}{2(n-1)b} \int \frac{dx}{(a + bx^2)^{n-1}} \quad (n \text{ integer} > 1)$$

$$\int \frac{dx}{x^2 (a + bx^2)^n} = \frac{1}{a} \int \frac{dx}{x^2 (a + bx^2)^{n-1}} - \frac{b}{a} \int \frac{dx}{(a + bx^2)^n}$$

(n positive integer)

Expressions involving $\sqrt{a + bx}$:

$$\int \sqrt{a + bx} \, dx = \frac{2}{3b} \sqrt{(a + bx)^3}$$

$$\int x \sqrt{a + bx} \, dx = -\frac{2(2a - 3bx)\sqrt{(a + bx)^3}}{15b^2}$$

$$\int x^2 \sqrt{a + bx} \, dx = \frac{2(8a^2 - 12abx + 15b^2x^2)\sqrt{(a + bx)^3}}{105b^3}$$

$$\int \frac{\sqrt{a + bx}}{x} dx = 2\sqrt{a + bx} + a \int \frac{dx}{x\sqrt{a + bx}}$$

$$\int \frac{dx}{\sqrt{a + bx}} = \frac{2\sqrt{a + bx}}{b}$$

$$\int \frac{x \, dx}{\sqrt{a + bx}} = -\frac{2(2a - bx)}{3b^2} \sqrt{a + bx}$$

$$\int \frac{x^2 \, dx}{\sqrt{a + bx}} = \frac{2(8a^2 - 4abx + 3b^2x^2)}{15b^3} \sqrt{a + bx}$$

$$\int \frac{dx}{x\sqrt{a + bx}} = \frac{1}{\sqrt{a}} \log \left[\frac{\sqrt{a + bx} - \sqrt{a}}{\sqrt{a + bx} + \sqrt{a}}\right] \quad (a \text{ pos.})$$

or

$$\int \frac{dx}{x\sqrt{a + bx}} = \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bx}{-a}} \quad (a \text{ neg.})$$

$$\int \frac{dx}{x^2 \sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{a+bx}}$$

$$\int \frac{dx}{(a+bx)(c+dx)} = \frac{1}{ad-bc}\log\frac{c+dx}{a+bx}$$

$$\int \frac{c+dx}{\sqrt{a+bx}}dx = \frac{2}{3b^2}(3bc - 2ad + bdx)\sqrt{a+bx}$$

$$\int \frac{\sqrt{a+bx}}{c+dx}dx = \frac{2\sqrt{a+bx}}{d}$$

$$-\frac{2}{d}\sqrt{\frac{bc-ad}{d}}\tan^{-1}\sqrt{\frac{d(a+bx)}{bc-ad}} \quad (d \text{ pos } bc > ad)$$

$$\int \frac{\sqrt{a+bx}}{c+dx}dx = \frac{2\sqrt{a+bx}}{d}$$

$$+\frac{1}{d}\sqrt{\frac{ad-bc}{d}}\log\left(\frac{\sqrt{d(a+bx)} - \sqrt{ad-bc}}{\sqrt{d(a+bx)} + \sqrt{ad-bc}}\right)$$

$$(d \text{ pos } ad > bc)$$

$$\int \frac{dx}{(c+dx)\sqrt{a+bx}}$$

$$= \frac{2}{\sqrt{d}\sqrt{bc-ad}}\tan^{-1}\sqrt{\frac{d(a+bx)}{bc-ad}} \quad (d \text{ pos } bc > ad)$$

$$\int \frac{dx}{(c+dx)\sqrt{a+bx}}$$

$$= \frac{1}{\sqrt{d}\sqrt{ad-bc}}\log\frac{\sqrt{d(a+bx)} - \sqrt{ad-bc}}{\sqrt{d(a+bx)} + \sqrt{ad-bc}}$$

$$(d \text{ pos } ad > bc)$$

Expressions involving $\sqrt{a^2 - x^2}$ or $\sqrt{a^2 + x^2}$:

$$\int \sqrt{a^2 - x^2}dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right]$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\left[\log \frac{a + \sqrt{a^2 \pm x^2}}{x}\right]$$

$$\int \frac{\sqrt{a^2 \pm x^2}}{x}\, dx = \sqrt{a^2 \pm x^2} - a \log\left[\frac{a + \sqrt{a^2 \pm x^2}}{x}\right]$$

$$\int \frac{x\, dx}{\sqrt{a^2 \pm x^2}} = \pm \sqrt{a^2 \pm x^2}$$

$$\int x\sqrt{a^2 - x^2}\, dx = -\tfrac{1}{3}\sqrt{(a^2 - x^2)^3}$$

$$\int \sqrt{(a^2 - x^2)^3}\, dx = \frac{x}{8}(5\, a^2 - 2\, x^2)\sqrt{a^2 - x^2} + \frac{3}{8} a^4 \sin^{-1}\frac{x}{a}$$

$$\int x^2\sqrt{a^2 - x^2}\, dx = -\frac{x}{4}\sqrt{(a^2 - x^2)^3} + \frac{a^2}{8}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right]$$

$$\int \frac{x^2\, dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

$$\int \frac{x^2\, dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x}} - \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x}$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2}\, dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1}\frac{x}{a}$$

Expressions involving $\sqrt{x^2 + a^2}$ or $\sqrt{x^2 - a^2}$:

$$\int \sqrt{x^2 \pm a^2}\, dx = \tfrac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \log\left(x + \sqrt{x^2 \pm a^2}\right)\right]$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left[x + \sqrt{x^2 \pm a^2}\right]$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \cos^{-1} \frac{a}{x}$$

$$\int \frac{\sqrt{x^2 - a^2}}{x}\, dx = \sqrt{x^2 - a^2} - a \cos^{-1} \frac{a}{x}$$

$$\int \frac{x\, dx}{\sqrt{x^2 - a^2}} = \sqrt{x^2 - a^2}$$

$$\int \frac{x\, dx}{\sqrt{(x^2 \pm a^2)^3}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

$$\int x\sqrt{x^2 \pm a^2}\, dx = \tfrac{1}{3}\sqrt{(x^2 \pm a^2)^3}$$

$$\int \sqrt{(x^2 \pm a^2)^3}\, dx = \frac{x}{8}(2\,x^2 \pm 5\,a^2)\sqrt{x^2 \pm a^2} + \frac{3\,a^4}{8}\log\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int x^2\sqrt{x^2 \pm a^2}\, dx = \frac{x}{8}(2\,x^2 \pm a^2)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\log\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{x^2\, dx}{\sqrt{x^2 \pm a^2}} = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{x^2\, dx}{\sqrt{(x^2 \pm a^2)^3}} = -\frac{x}{\sqrt{x^2 \pm a^2}} + \log\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

$$\int \frac{\sqrt{x^2 \pm a^2}\,dx}{x^2} = -\frac{\sqrt{x^2 \pm a^2}}{x} + \log\left(x + \sqrt{x^2 \pm a^2}\right)$$

Expressions involving $ax^2 + bx + c$.

$$\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2-4ac}} \log \frac{(2ax+b) - \sqrt{b^2-4ac}}{(2ax+b) + \sqrt{b^2-4ac}}$$

if $b^2 > 4ac$

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}$$

if $(b^2 < 4ac)$

$$\int \frac{dx}{ax^2+bx+c} = \frac{-2}{2ax+b} \qquad \text{if } b^2 = 4ac$$

$$\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a} \log(ax^2+bx+c) - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c}$$

$$\int \frac{x^2\,dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2} \log(ax^2+bx+c) + \frac{b^2-2ac}{2a^2} \int \frac{dx}{ax^2+bx+c}$$

Expressions involving $\sqrt{\pm ax^2 + bx + c}$:

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \log\left(2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right)$$

$$\int \sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} - \frac{b^2-4ac}{8a} \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int \frac{dx}{\sqrt{-ax^2+bx+c}} = \frac{1}{\sqrt{a}} \sin^{-1}\left(\frac{2\,ax-b}{\sqrt{b^2+4\,ac}}\right)$$

$$\int \sqrt{-ax^2+bx+c}\,dx = \frac{2\,ax-b}{4\,a}\sqrt{-ax^2+bx+c}$$

$$+\frac{b^2+4\,ac}{8\,a}\int \frac{dx}{\sqrt{-ax^2+bx+c}}$$

$$\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}} = -\frac{2\,(2\,ax+b)}{(b^2-4\,ac)\sqrt{ax^2+bx+c}}$$

Formulæ involving $\sqrt{2\,ax-x^2}$:

$$\int \sqrt{2\,ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2\,ax-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a}$$

$$\int x\sqrt{2\,ax-x^2}\,dx = -\frac{3\,a^2+ax-2\,x^2}{6}\sqrt{2\,ax-x^2}$$

$$+\frac{a^3}{2}\operatorname{vers}^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{2\,ax-x^2}} = \operatorname{vers}^{-1}\frac{x}{a}$$

$$\int \frac{x\,dx}{\sqrt{2\,ax-x^2}} = -\sqrt{2\,ax-x^2} + a\operatorname{vers}^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{2\,ax-x^2}} = -\frac{\sqrt{2\,ax-x^2}}{ax}$$

$$\int \frac{\sqrt{2\,ax-x^2}}{x}\,dx = \sqrt{2\,ax-x^2} + a\operatorname{vers}^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{(2\,ax-x^2)^3}} = \frac{x-a}{a^2\sqrt{2\,ax-x^2}}$$

$$\int \sqrt{\frac{a+x}{b+x}}\,dx = \sqrt{(a+x)\,(b+x)}$$

$$+(a-b)\log\left[\sqrt{a+x}+\sqrt{b+x}\right]$$

$$\int \sqrt{\frac{a-x}{b+x}}\,dx = \sqrt{(a-x)\,(b+x)} + (a+b)\sin^{-1}\sqrt{\frac{b+x}{a+b}}$$

$$\int \frac{dx}{x(a+bx^n)} = \frac{1}{an}\log\frac{x^n}{a+bx^n}$$

$$\int \frac{dx}{x\sqrt{a+bx^n}} = \frac{1}{n\sqrt{a}}\log\frac{\sqrt{(a+bx^n)}-\sqrt{a}}{\sqrt{a+bx^n}+\sqrt{a}} \qquad (a \text{ pos.})$$

$$\int \frac{dx}{x\sqrt{a+bx^n}} = \frac{2}{n\sqrt{-a}}\sec^{-1}\sqrt{\frac{-bx^n}{a}} \qquad (a \text{ neg.})$$

Expressions involving trigonometric forms:

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

$$\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \sin x\cos x\,dx = \tfrac{1}{2}\sin^2 x$$

$$\int \sin^2 x\cos^2 x\,dx = -\tfrac{1}{8}\left[\tfrac{1}{4}\sin(4x) - x\right]$$

$$\int \sin x\cos^m x\,dx = -\frac{\cos^{m+1} x}{m+1}$$

$$\int \sin^m x\cos x\,dx = \frac{\sin^{m+1} x}{m+1}$$

$$\int \cos^m x\sin^n x\,dx = \frac{\cos^{m-1} x\sin^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \cos^{m-2} x\sin^n x\,dx$$

$$\int \cos^m x \sin^n x\, dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n}\int \cos^m x \sin^{n-2} x\, dx$$

$$\int \frac{\sin^m x}{\cos^n x}\, dx = \frac{\sin^{m+1} x}{(n-1)\cos^{n-1} x} + \frac{n-m-2}{n-1}\int \frac{\sin^m x}{\cos^{n-2} x}\, dx$$

$$\int \frac{\cos^n x}{\sin^m x}\, dx = -\frac{\cos^{n+1} x}{(m-1)\sin^{m-1} x} + \frac{m-n-2}{m-1}\int \frac{\cos^n x}{\sin^{m-2} x}\, dx$$

$$\int \frac{dx}{\sin^m x} = -\frac{\cos x}{(m-1)\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{dx}{\sin^{m-2} x}$$

$$\int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x}$$

$$\int \tan x\, dx = -\log \cos x$$

$$\int \tan^2 x\, dx = \tan x - x$$

$$\int \cot x\, dx = \log \sin x$$

$$\int \cot^2 x\, dx = -\cot x - x$$

$$\int \sec x\, dx = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \frac{1}{2}\log\frac{1+\sin x}{1-\sin x}$$

$$\int \sec^2 x\, dx = \tan x$$

$$\int \operatorname{cosec} x\, dx = \log \tan\left(\tfrac{1}{2} x\right)$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x$$

$$\int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)$$

$$\int \frac{dx}{1 - \sin ax} = \frac{1}{a} \cot\left(\frac{\pi}{4} - \frac{ax}{2}\right)$$

$$\int \frac{dx}{b+c \sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1}\left[\sqrt{\frac{b-c}{b+c}} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right] \qquad b^2 > c^2$$

$$\int \frac{dx}{b + c \sin ax} = -\frac{1}{a\sqrt{c^2 - b^2}} \log\left(\frac{c + b \sin ax + \sqrt{c^2 - b^2} \cos ax}{b + c \sin ax}\right) \qquad c^2 > b^2$$

$$\int \sin ax \sin bx \, dx = \frac{\sin (a - b) x}{2 (a - b)} - \frac{\sin (a + b) x}{2 (a + b)} \qquad (a^2 \neq b^2)$$

$$\int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2}$$

$$\int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2}$$

$$\int \frac{dx}{b + c \cos ax} = \frac{2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left(\sqrt{\frac{b - c}{b + c}} \tan \frac{ax}{2}\right) \qquad b^2 > c^2$$

$$\int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{c^2 - b^2}} \log\left(\frac{c + b \cos ax + \sqrt{c^2 - b^2} \sin ax}{b + c \cos ax}\right) \qquad c^2 > b^2$$

$$\int \cos ax \cos bx \, dx = \frac{\sin (a-b) x}{2 (a-b)} + \frac{\sin (a + b) x}{2 (a + b)} \qquad a^2 \neq b^2$$

$$\int \sin ax \cos bx \, dx = -\frac{1}{2}\left[\frac{\cos(a-b)x}{a-b} + \frac{\cos(a+b)x}{a+b}\right]$$
$$a \neq b^2$$

$$\int \frac{dx}{b \sin ax + c \cos ax}$$
$$= \frac{1}{a\sqrt{b^2+c^2}} \log\left[\tan\frac{1}{2}\left(ax + \tan^{-1}\frac{c}{b}\right)\right]$$

$$\int x \sin x \, dx = \sin x - x \cos x$$

$$\int x^2 \sin x \, dx = 2x \sin x - (x^2 - 2)\cos x$$

$$\int x \cos x \, dx = \cos x + x \sin x$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2)\sin x$$

$$\int \frac{\sin ax \, dx}{x} = ax - \frac{(ax)^3}{3\,\underline{|3}} + \frac{(ax)^5}{5\,\underline{|5}} - \ldots$$

$$\int \frac{\cos ax \, dx}{x} = \log ax - \frac{(ax)^2}{2\,\underline{|2}} + \frac{(ax)^4}{4\,\underline{|4}} - \ldots$$

Transcendentals

$$\int \log x \, dx = x \log x - x$$

$$\int \frac{(\log x)^n}{x} dx = \frac{1}{n+1}(\log x)^{n+1}$$

$$\int \frac{dx}{x \log x} = \log \log x$$

$$\int \frac{dx}{x(\log x)^n} = -\frac{1}{(n-1)(\log x)^{n-1}}$$

$$\int x^m \log x \, dx = x^{m+1}\left[\frac{\log x}{m+1} - \frac{1}{(m+1)^2}\right]$$

$$\int x\, e^{ax}\, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^m e^{ax}\, dx = \frac{x^m e^{ax}}{a} - \frac{m}{a}\int x^{m-1} e^{ax}\, dx$$

$$\int \frac{e^{ax}}{x^m}\, dx = -\frac{1}{m-1}\,\frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1}\int \frac{e^{ax}}{x^{m-1}}\, dx$$

$$\int e^{ax} \sin(nx)\, dx = e^{ax}\left[\frac{a \sin(nx) - n \cos(nx)}{a^2 + n^2}\right]$$

$$\int e^{ax} \cos(nx)\, dx = e^{ax}\left[\frac{a \cos(nx) + n \sin(nx)}{a^2 + n^2}\right]$$

HYPERBOLIC FUNCTIONS

Hyperbolic Transformations

$$\sinh x = \frac{e^x - e^{-x}}{2} = -j \sin(jx)$$

where $$j = \sqrt{-1}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \cos(jx)$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = -j \tan(jx)$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = j \cot(jx)$$

$$e^x = \cosh x + \sinh x$$

$$e^{-x} = \cosh x - \sinh x$$

$$\sin x = -j \sinh(jx)$$

$$\cos x = \cosh(jx)$$

Hyperbolic Formulæ

$$\cosh^2 x - \sinh^2 x = 1$$

$$\operatorname{sech}^2 x + \tanh^2 x = 1$$

$$\coth^2 x - \operatorname{cosech}^2 x = 1$$

$$\sinh (x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh (x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\sinh (x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cosh (x - y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\tanh (x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$

$$\coth (x + y) = \frac{\coth x \coth y + 1}{\coth y + \coth x}$$

$$\tanh (x - y) = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$$

$$\coth (x - y) = \frac{\coth x \coth y - 1}{\coth y - \coth x}$$

$$\sinh (2x) = 2 \sinh x \cosh x$$

$$\cosh (2x) = \cosh^2 x + \sinh^2 x$$

$$\tanh (2x) = \frac{2 \tanh x}{1 + \tanh^2 x}$$

$$\coth (2x) = \frac{\coth^2 x + 1}{2 \coth x}$$

$$\sinh \left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x - 1}{2}}$$

$$\cosh \left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x + 1}{2}}$$

$$\tanh \left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$$

$$\coth \left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x + 1}{\cosh x - 1}}$$

$$\sinh x + \sinh y = 2 \sinh \left(\frac{x + y}{2}\right) \cosh \left(\frac{x - y}{2}\right)$$

$$\sinh x - \sinh y = 2 \cosh\left(\frac{x+y}{2}\right) \sinh\left(\frac{x-y}{2}\right)$$

$$\cosh x + \cosh y = 2 \cosh\left(\frac{x+y}{2}\right) \cosh\left(\frac{x-y}{2}\right)$$

$$\cosh x - \cosh y = 2 \sinh\left(\frac{x+y}{2}\right) \sinh\left(\frac{x-y}{2}\right)$$

$$\sinh(3x) = 3 \sinh x + 4 \sinh^3 x$$

$$\cosh(3x) = -3 \cosh x + 4 \cosh^3 x$$

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \log\left(x + \sqrt{1 + x^2}\right)$$

$$\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$$

$$\tanh^{-1} x = \frac{1}{2} \log\left[\frac{1+x}{1-x}\right]$$

$$\coth^{-1} x = \frac{1}{2} \log\left[\frac{x+1}{x-1}\right]$$

$$\text{sech}^{-1} x = \log\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$$

$$\text{cosech}^{-1} x = \log\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)$$

Differentials of Hyperbolic Functions

$$d(\sinh x) = \cosh x\, dx$$

$$d(\cosh x) = \sinh x\, dx$$

$$d(\tanh x) = \text{sech}^2 x\, dx$$

$$d(\coth x) = -\text{cosech}^2 x\, dx$$

$$d(\text{sech}\, x) = -\text{sech}\, x \tanh x\, dx$$

$$d(\text{cosech}\, x) = -\text{cosech}\, x \coth x\, dx$$

$$d\,(\sinh^{-1} x) = \frac{dx}{\sqrt{1 + x^2}}$$

$$d\,(\cosh^{-1} x) = \frac{dx}{\sqrt{x^2 - 1}}$$

$$d\,(\tanh^{-1} x) = \frac{dx}{1 - x^2}$$

$$d\,(\coth^{-1} x) = \frac{dx}{1 - x^2}$$

$$d\,(\operatorname{sech}^{-1} x) = -\frac{dx}{x\sqrt{1 - x^2}}$$

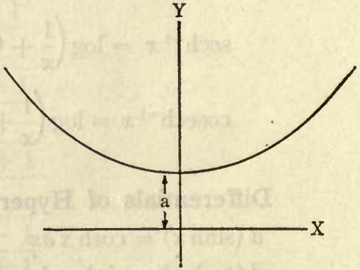
$$d\,(\operatorname{cosech}^{-1} x) = -\frac{dx}{x\sqrt{x^2 + 1}}$$

Use of Hyperbolic Functions

Illustrative Example. Deduce an expression for the length of a perfectly flexible chain suspended between two supports; assume that both points of support are the same height from the ground.

The chain assumes the form of a catenary (see page 33), the equation of which is

$$y = a \cosh \frac{x}{a}$$



The general equation for the length of the chain is

$$L = \text{length} = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

where the value of $\frac{dy}{dx}$, obtained by differentiating the equation of the catenary, is

$$\frac{dy}{dx} = \frac{d\left(a \cosh \frac{x}{a}\right)}{dx} = a\left[\left(\sinh \frac{x}{a}\right)\left(\frac{1}{a}\right)\right] = \sinh \frac{x}{a}$$

Substituting the value of $\frac{dy}{dx}$ in the formula for the length, L, we have

$$L = \int \sqrt{1 + \sinh^2 \frac{x}{a}}\, dx = \int \sqrt{\cosh^2 \frac{x}{a}}\, dx$$

$$= \int \cosh \frac{x}{a}\, dx = a \sinh \frac{x}{a}$$

which is the required expression for the length of the chain.

DIFFERENTIAL EQUATIONS

A **differential equation** is a relation involving derivatives or differentials.

A **solution** of a differential equation is a relation between the variables which satisfies the given equation.

ORDINARY DIFFERENTIAL EQUATIONS

Equations of the **First Order** and **First Degree**

I. An equation of the form

$$f_1(x)\, dx + f_2(y)\, dy = 0$$

can be integrated immediately.

Its solution is

$$\int f_1(x)\, dx + \int f_2(y)\, dy = C$$

An equation may sometimes be changed to the above form by separation of the variables.

II. **Homogeneous Equation.** An equation is **homogeneous** in respect to its variables when the

sum of their exponents is the same for each term of the equation.

Homogeneous equations are reduced to the form of Method I, by substituting vx for y, and then separating the variables.

III. Non-homogeneous Equation of First Degree in x and y. This type occurs in the form:

$$(ax + by + c)\,dx = (a'x + b'y + c')\,dy$$

Substitute for x, $(x' + h)$, and for y, $(y' + k)$. The equation then becomes:

$$(ax' + by' + ah + bk + c)dx' = (a'x' + b'y' + a'h + b'k + c')\,dy'$$

Equating $\qquad ah + bk + c = 0$

and $\qquad a'h + b'k + c' = 0$

the original equation now takes the form:

$$(ax' + by')\,dx' = (a'x' + b'y')\,dy'$$

which is homogeneous and solvable by Method **II.**

In the solution thus obtained, substitute

$$x' = x - h \qquad \text{and} \qquad y' = y - k$$

where h and k are determined from the two equations:

$$ah + bk + c = 0$$
$$a'h + b'k + c' = 0$$

IV. Linear Equation. A linear differential equation (of first order and first degree) is of the general form:

$$\frac{dy}{dx} + Py = Q$$

where P and Q are functions of x alone or constants.

The solution of this equation is

$$ye^{\int P\,dx} = \int e^{\int P\,dx}\,Q\,dx + C$$

V. Equations Reducible to the Linear Equation. This type occurs in the form:

$$\frac{dy}{dx} + Py = Qy^n$$

where P and Q are functions of x alone. The given equation may be written:

$$\frac{dv}{dx} + (1 - n) Pv = (1 - n) Q$$

where $v = y^{-n+1}$. This equation is linear in v, and solvable by Method IV. In the solution, resubstitute for v its value y^{-n+1}.

VI. Exact Differential Equation. An equation of the form

$$M\,dx + N\,dy = 0$$

is exact if the derivative of M with regard to y is equal to the derivative of N with regard to x. The solution then is:

$$\int M\,dx + \int \left[N - \frac{\partial}{\partial y}\int M\,dx\right] dy = C$$

where $\int M\,dx$ is the integral of M with respect to x (regarding y as constant), and the term

$$\left[N - \frac{\partial}{\partial y}\int M\,dx\right]$$

is found by subtracting from N the derivative in respect to y of $\int M\,dx$. The term $\left[N - \frac{\partial}{\partial y}\int M\,dx\right]$ is integrated with regard to y (considering x constant). The complete solution is then given by the formula above.

VII. Integrating Factors. If a differential equation of the form

$$M\,dx + N\,dy = 0$$

is multiplied through by a certain expression called an integrating factor, the equation will become exact. It is then solvable by Method VI.

(a) When an equation is homogeneous, $\dfrac{1}{Mx+Ny}$ is an integrating factor.

(b) When the condition exists that

$$\frac{\dfrac{dM}{dy} - \dfrac{dN}{dx}}{N} = F(x) \quad \text{[an expression containing only } x\text{]}$$

then $e^{\int F(x)\,dx}$ is an integrating factor.

(c) Similarly when

$$\frac{\dfrac{dN}{dx} - \dfrac{dM}{dy}}{M} = F(y)$$

then $e^{\int F(y)\,dy}$ is an integrating factor.

Equations of the First Order but Higher than the First Degree

In the following formulæ, $\dfrac{dy}{dx}$ will be denoted by p.

An equation of first order and of nth degree is of the general form

$$p^n + Ap^{n-1} + Bp^{n-2} + \cdots + Jp + K = 0$$

where the coefficients $A, B, \cdots J, K$ are functions of x and y.

I. Clairaut's Equation. When an equation is of the form

$$y = px + f(p)$$

the solution is obtained by substituting for p a constant c,

$$y = cx + f(c)$$

II. Solution by Factoring. The given equation may sometimes be resolved into rational factors of the first degree. Each factor is equated separately to zero, and its solution found by one of the preceding methods, using the same constant of integration in each case. The complete solution is then the product of the separate solutions.

III. **Equations Containing only x and p.** When an equation is of this type, solve for p, and substitute its value $\frac{dy}{dx}$. The resulting equation can be integrated immediately.

IV. **Equations Containing only y and p.** Solve for p, and substitute its value $\frac{dy}{dx}$. This equation is immediately integrable.

V. **Equations Involving x, y, and p.** A solution can be obtained by one of the following methods:

(a) Solve for x in terms of y and p. Then differentiate in respect to y, remembering that $\frac{dx}{dy} = \frac{1}{p}$.

The solution of this equation, together with the given equation, constitutes the complete solution.

(b) Solve for y in terms of x and p. Differentiate with respect to x, and in place of $\frac{dy}{dx}$ substitute its value

p. The complete solution consists of the solution of this equation, together with the original equation.

(c) Solve for p, and replace it with its value $\frac{dy}{dx}$. From this equation it may be possible to obtain a solution.

Linear Differential Equations with Constant Coefficients

A **linear differential equation** is of the first degree in the dependent variable and all of its derivatives.

The **particular integral** is the solution of the equation obtained without the introduction of constants of integration.

The **complementary function** is the solution obtained by temporarily equating to zero all those terms of the equation that do not contain the dependent variable or derivatives thereof.

The **complete solution** is the sum of the particular integral and the complementary function.

A linear equation with **constant coefficients** is of the form:

$$\frac{d^n y}{dx^n} + P\frac{d^{n-1}y}{dx^{n-1}} + Q\frac{d^{n-2}y}{dx^{n-2}} + \cdots + Ry = X$$

where the coefficients $P, Q, \cdots R$ are constants; and X is a function of x. Replacing $\frac{d}{dx}$ by the symbol D, the equation becomes

$$(D^n + PD^{n-1} + QD^{n-2} + \cdots + R)\,y = X.$$

Case I. Method of Solution when X = 0. Write the given integral in its symbolic form, replacing $\frac{d}{dx}$

by D. Then solve this equation for D as if it were an ordinary algebraic quantity.

When the **roots** of the equation (i.e., the values of D) are **real,** the solution is

$$y = c_1e^{m_1x} + c_2e^{m_2x} + \cdots$$

where c_1, c_2, etc., are the constants of integration, and m_1, m_2, etc., are the roots of the equation.

When two or more real roots of the equation are **equal,** the solution is

$$y = (c_1 + c_2x + c_3x^2 + \cdots)\, e^{mx} + \cdots$$

where m is the value of the repeated root, and c_1, c_2, c_3, etc., are the constants of integration (introduced in the manner shown in the above equation) and equal in number to the number of times the root m is repeated.

When the equation has **imaginary roots** (which always occur in pairs) the solution is

$$y = e^{m_1x}\,[A \cos(a_1x) + B \sin(a_1x)] + e^{m_2x}\,[C \cos(a_2x) + D \sin(a_2x)] + \cdots$$

where A and B, C and D, etc., are the constants of integration, and $(m_1 \pm a_1\sqrt{-1})$, $(m_2 \pm a_2\sqrt{-1})$, etc., are the complex imaginary roots of the equation.

When two or more pairs of complex imaginary roots are **equal,** the solution is

$$y = [(c_1 + c_2x + \cdots) \cos(ax) + (c_3 + c_4x + \cdots) \sin(ax)]\, e^{mx}$$

where $(m \pm a\sqrt{-1})$ is the repeated pair of complex imaginary roots.

Case II. Method of solution when X is not equal to zero. In this case, the **complete solution** is

the **sum** of the complementary function and the particular integral.

The **complementary function** is found by temporarily equating $X = 0$, and obtaining the solution by the method of **Case I.**

The **particular integral** is obtained as follows.

The given equation is of the general form:

$$(D^n + PD^{n-1} + QD^{n-2} + \cdots + R)\, y = X$$

in which D is used in place of $\frac{d}{dx}$.

In symbolic notation, this equation may be expressed

$$f(D)\, y = X$$

The particular integral can then be written:

$$y = \frac{X}{f(D)} = \text{particular integral}$$

A. Method of obtaining the particular integral when the term X is of the form e^{ax}.

$$\text{particular integral} = \frac{X}{f(D)} = \frac{e^{ax}}{f(D)} = \frac{e^{ax}}{f(a)}$$

which is found by substituting the constant a in place of D.

This method for evaluating $\frac{e^{ax}}{f(D)}$ fails when the term $(D - a)$ is a factor of $f(D)$. The particular integral is then found by substituting the constant a for D in all terms of $f(D)$ except in the factor $(D - a)$. The solution is then completed by the general method given under case F (page 84).

B. Solution for the particular integral when X has the form x^m.

$$\text{particular integral} = \frac{X}{f(D)} = \frac{x^m}{f(D)} = [f(D)]^{-1} x^m$$

To evaluate this expression, expand $[f(D)]^{-1}$ into a series of ascending powers of D, by use of the binomial theorem. It is only necessary to carry out this expansion to the mth power of D, since operation on x^m by higher powers of D would produce zero (since the symbol D stands for $\frac{d}{dx}$, the operation by D on a quantity denotes its derivative with respect to x, the operation by D^2 denotes its second derivative, etc.). In obtaining the solution of the given particular integral, x^m is operated on separately by each term of the expansion of $[f(D)]^{-1}$.

C. **Method of obtaining the particular integral when X has the form sin (ax).**

$$\text{particular integral} = \frac{X}{f(D)} = \frac{\sin(ax)}{f(D)}$$

In order to evaluate this integral, substitute $-a^2$ for D^2 wherever D^2 occurs in $f(D)$. The particular integral will then be a fraction, whose numerator is $\sin(ax)$, and whose denominator is the value assumed by $f(D)$ when D^2 is replaced by $-a^2$.

This method fails if $f(D)$ becomes zero when $-a^2$ is substituted for D^2. The particular integral is then evaluated by writing the term e^{iax} (in which $i = \sqrt{-1}$) in place of $\sin(ax)$. The solution of this new integral is obtained by method A for the evaluation of the particular integral. In the result, e^{iax} is replaced by $[\cos(ax) + i\sin(ax)]$, producing a result containing both real and imaginary terms. The required particular integral is the coefficient of i (i.e., $\sqrt{-1}$) in this expression.

D. **Particular Integral when X = cos (ax).** The

particular integral is obtained as in method C, with the exception that cos (ax) is used in place of sin (ax).

When this method fails, e^{iax} is written in place of cos (ax), and this new integral is evaluated by method A. In the solution of this integral, e^{iax} is replaced by $[\cos(ax) + i \sin(ax)]$. The required particular integral is the real part of this result.

E. Particular integral when X is of the form $e^{ax}Q$.

$$\text{particular integral} = \frac{X}{f(D)} = \frac{e^{ax}Q}{f(D)} = e^{ax}\frac{Q}{f(D+a)}$$

To evaluate the given integral, $(D + a)$ is substituted for D, wherever D occurs in $f(D)$; and the term e^{ax} is treated as a constant multiplier. The new integral $\dfrac{Q}{f(D+a)}$ is evaluated by one of the preceding methods, or by the general method F. The required particular integral is then equal to the product of e^{ax} by the evaluation of $\dfrac{Q}{f(D+a)}$.

F. General method for finding the particular integral.

To evaluate $\dfrac{1}{f(D)}X$.

The denominator of $\dfrac{1}{f(D)}$ may be resolved into factors of the first degree. The given integral then becomes:

$$\frac{1}{(D-a)}\,\frac{1}{(D-b)}\,\frac{1}{(D-c)}\,\frac{1}{(D-d)}\cdots\frac{1}{(D-m)}X$$

The term X is operated on successively by each of these fractional operators, beginning at the right. The

operation on X by the first factor $\frac{1}{(D-m)}$ produces the expression $e^{mx}\int e^{-mx}X\,dx$. This result is operated on in a similar manner by each remaining factor (proceeding from right to left). The solution of the given particular integral is then:

$$e^{ax}\int e^{-ax}e^{bx}\int e^{-bx}e^{cx}\int e^{-cx}\ldots e^{mx}\int e^{-mx}X(dx)^m$$

Homogeneous Linear Equation

The homogeneous linear equation is of the form

$$x^n\frac{d^ny}{dx^n}+Px^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+\cdots+Ry=X$$

in which the coefficients $P, \ldots R$ are constants, and X is a function of x.

On assuming the relation, $x = e^z$, this equation may be transformed by the substitutions:

$$x^n\frac{d^ny}{dx^n}=\theta(\theta-1)(\theta-2)\cdots \text{ to } n \text{ terms}$$

$$x^{n-1}\frac{d^{n-1}y}{dx^{n-1}}=(\theta-1)(\theta-2)(\theta-3)\cdots\text{ to } (n-1) \text{ terms},$$

and so forth; where the symbol θ stands for $\frac{d}{dz}\cdot$

The **complementary function** is then found as in the case of the linear equation with constant coefficients. (In obtaining this solution, the term θ is treated in exactly the same manner in which the term D was treated in the preceding cases.)

In order to obtain the **particular integral,** the term X (which involves only x) is changed to an expression

involving z, by the substitution $x = e^z$. The particular integral is then found by one of the methods given under the case of the linear equation with constant coefficients.

The **complete solution** is the sum of the complementary function and the particular integral. In the result, z is replaced by its value $\log x$.

Exact Differential Equations

An exact differential equation is one which can be derived directly by differentiation of an equation of the next lower order.

If the given equation is of the form:

$$A\frac{d^ny}{dx^n} + B\frac{d^{n-1}y}{dx^{n-1}} + \cdots + Q\frac{d^3y}{dx^3} + R\frac{d^2y}{dx^2} + S\frac{dy}{dx} + Ty = X$$

where $A, B, \ldots Q, R, S, T$, and X are functions of x, we then have as the condition for exactness that:

$$T - \frac{dS}{dx} + \frac{d^2R}{dx^2} - \frac{d^3Q}{dx^3} + \cdots = 0$$

The first integral of the given equation then is:

$$A\frac{d^{n-1}y}{dx^{n-1}} + \left(B - \frac{dA}{dx}\right)\frac{d^{n-2}y}{dx^{n-2}} + \left(C - \frac{dB}{dx} + \frac{d^2A}{dx^2}\right)\frac{d^{n-3}y}{dx^{n-3}} \cdots = \int X\,dx + C$$

This formula may be reapplied successively as long as each resulting equation satisfies the condition for exactness.

Equations of the Second Order and the First Degree

General form is

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$$

where P, Q, and X are functions of x.

I. **When one solution of the equation is known (or can be found by inspection).**

Let y_1 equal the known integral. In the given equation, substitute vy_1 in place of y; and then, in the transformed equation, replace $\frac{dv}{dx}$ by p. This equation can be solved by one of the preceding methods.

II. **Change of the Independent Variable.**

The purpose of this change and of the removal of the first derivative (see III) is to transform a given equation into a new equation which may happen to be easily integrable.

The given equation is of the form:

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$$

By changing the independent variable, it may be transformed into the following equation:

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1y = X_1$$

where Q_1 becomes equal to 1, if

$$\frac{dz}{dx} = \sqrt{Q}$$

when also $$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{Q}$$

and $$X_1 = \frac{X}{Q}$$

or where P_1 may be made equal to zero, if

$$z = \int e^{-\int P\,dx}\,dx$$

when also

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}$$

and

$$X_1 = \frac{X}{\left(\frac{dz}{dx}\right)^2}$$

III. Removal of the First Derivative.

To remove the first derivative from an equation of the form

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$$

make the substitution $y = ve^{-\frac{1}{2}\int P\,dx}$

The given equation then becomes

$$\frac{d^2v}{dx^2} + Q_1 v = X_1$$

where

$$Q_1 = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$$

and

$$X_1 = Xe^{\frac{1}{2}\int P\,dx}$$

THEORETICAL MECHANICS

Center of Gravity

The **center of gravity** of a body is a point so situated that the force of gravity produces no tendency in the body to rotate about any axis passing through this point.

Center of Gravity of the Arc of a Plane Curve

$$\bar{x} = \frac{\int x\,ds}{\int ds} = \frac{\int x\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx}{\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx}$$

$$\bar{y} = \frac{\int y\,ds}{\int ds} = \frac{\int y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy}{\int\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy}$$

where $\bar{x}$ and $\bar{y}$ are the coördinates of the center of gravity.

Solve for y in terms of x from the equation of the given curve. Then differentiate in order to obtain $\frac{dy}{dx}$, and substitute its value in the formula for $\bar{x}$.

Similarly, find x in terms of y, obtain $\frac{dx}{dy}$, and substitute in the formula for $\bar{y}$.

Center of Gravity of Plane Areas

Rectangular Coördinates:

$$\bar{x} = \frac{\int\int x\,dA}{\int\int dA} = \frac{\int\int x\,dx\,dy}{\int\int dx\,dy}$$

$$\bar{y} = \frac{\int\int y\,dA}{\int\int dA} = \frac{\int\int y\,dx\,dy}{\int\int dx\,dy}$$

where $\bar{x}$ and $\bar{y}$ are the coördinates of the center of gravity.

In evaluating the expression for $\bar{x}$, we may integrate first either in respect to x or y, according to which method is more convenient.

If dy is integrated first, the limits of y are expressed in terms of x (from the given equation); and the limits of x are its initial and final values.

Similarly, if dx is first integrated, the limits of x are expressed in terms of y; and the limits of y are then its initial and final values.

Polar Coördinates:

$$\bar{x} = \frac{\int\int r^2 \cos\theta \, d\theta \, dr}{\int\int r \, d\theta \, dr}$$

$$\bar{y} = \frac{\int\int r^2 \sin\theta \, d\theta \, dr}{\int\int r \, d\theta \, dr}$$

Generally, it is more convenient to integrate first with respect to r. In this case, the limits of r are found in terms of θ from the equation of the given curve. The limits of θ are its initial and final values, expressed in radians.

Center of Gravity of Solids of Revolution. When a solid of uniform density is formed by the revolution of a plane curve about the X-axis, the center of gravity is on the X-axis (because of symmetry). Its x-coördinate is

$$\bar{x} = \frac{\int\int xy \, dx \, dy}{\int\int y \, dx \, dy}$$

where the limits are found as in the case of plane areas.

When a solid is formed by the revolution of a plane figure about the Y-axis, the y-coördinate of its center of gravity is

$$\bar{y} = \frac{\int\int xy \, dx \, dy}{\int\int x \, dx \, dy}$$

Center of Gravity of Any Section Composed of Two or More Simple Plane Figures

In order to find the center of gravity of such figures as tee-bars, channels, rails, etc., divide them up into their component rectangles or triangles. Then, obtain the center of gravity and the area of each separate figure. Choose any convenient axis in the plane of the given section and find the turning moment of each figure about this axis. Each turning moment is the product of the area of the figure by the distance from its center of gravity to the chosen axis. The sum of all these separate turning moments gives the turning moment of the total figure. On dividing this total moment by the total area of the figure, we obtain the distance from the chosen axis to the center of gravity of the figure. Care must be used, if the chosen axis passes through the given figure, to take distances on one side of this axis as positive, and on the other side as negative.

Generally, one coördinate of the center of gravity can be determined by the symmetry of the given section. When the figure is unsymmetrical, it may be necessary to take moments about two different axes in order to locate the center of gravity.

Moment of Inertia of Plane Areas

The **moment of inertia of a plane figure** about any given axis is equal to the integral of the product of each elementary area of the figure by the square of its distance from the axis.

Rectangular Moment of Inertia:

The **rectangular moment of inertia** of a plane figure

is its moment of inertia about any axis in the plane of the figure. The rectangular moment of inertia of a plane area about the X-axis is

$$I_x = \int\int y^2\,dx\,dy$$

The rectangular moment of inertia of a plane area about the Y-axis is

$$I_y = \int\int x^2\,dx\,dy$$

In either case, the limits of the variable first integrated are expressed in terms of the other variable.

The **moment of inertia** of a plane figure **about the gravity axis** (I_g) is its rectangular moment of inertia about any axis in the plane of the figure, passing through its center of gravity.

The **moment of inertia** of a plane figure **about any axis parallel to the gravity axis** and in the plane of the figure is equal to (I_g) plus the product of the area of the figure by the square of the distance between the two axes, thus:

$$I = I_g + Fd^2$$

Polar Moment of Inertia:

The **polar moment of inertia** (I_p) is the moment of inertia about any axis perpendicular to the plane of the given figure.

It is equal to the sum of the rectangular moments of inertia about two mutually perpendicular axes in the plane of the figure, passing through the foot of the polar axis.

In **rectangular coördinates,** the polar moment of inertia equals

$$I_p = I_x + I_y = \int\int (x^2 + y^2)\,dx\,dy$$

In **polar coördinates,** the formula for the polar moment of inertia is

$$I_p = \int\int R^3\,dR\,d\theta$$

It is generally more convenient to integrate first with respect to R, expressing its limits in terms of θ. The limits of θ are then its initial and final values.

Moment of Inertia of Solids

The moment of inertia of a solid (with center at origin) about the X-axis is

$$I = m\int\int\int (y^2 + z^2)\,dx\,dy\,dz$$

where m is the density, that is, the mass per unit volume.

Radius of Gyration

The **center of gyration** is that point in a revolving body at which, if the entire mass of the body were concentrated, the moment of inertia about the axis of rotation would be the same as that of the body.

The **radius of gyration,** k, is the distance from the axis of rotation to the center of gyration.

For plane sections, $k = \sqrt{\dfrac{I}{A}}$

For solids, $k = \sqrt{\dfrac{I}{M}} = \sqrt{\dfrac{I}{\left(\dfrac{W}{g}\right)}}$

in which k = radius of gyration,

I = the moment of inertia about the axis of rotation,

A = area of section,

M = mass of body,

W = weight of body.

Center of Percussion

The **center of percussion** or oscillation of a pendulum or other body vibrating or rotating about a fixed axis or center is that point at which, if the entire weight of the body were concentrated, the body would continue to vibrate in the same intervals of time.

The **radius of oscillation** is

$$h = \frac{I}{Md} = \frac{I}{\left(\frac{W}{g}\right)d}$$

in which I = the moment of inertia of body about axis of rotation,

d = distance from center of gravity of body to the axis of rotation,

h = distance from center of percussion or oscillation to the axis of rotation,

M = mass of body,

W = weight of body.

Motion of a Body

$$\text{velocity at any instant} = v = \frac{ds}{dt}$$

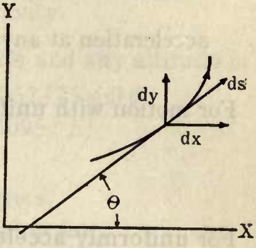
$$\text{acceleration at any instant} = a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

In rectangular coördinates,

$$v_x = \frac{dx}{dt} = \frac{ds}{dt}\cos\theta = \text{velocity in a direction parallel to the } X\text{-axis}$$

$$v_y = \frac{dy}{dt} = \frac{ds}{dt}\sin\theta$$

$$v = \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$



For motion with **uniform velocity,**

$$v = \frac{s}{t}$$

For **uniformly accelerated motion,**

$$s = \tfrac{1}{2}(u + v)\,t$$
$$s = ut + \tfrac{1}{2}at^2$$
$$2as = v^2 - u^2$$

u = initial velocity,
v = final velocity,
a = constant acceleration,
s = space passed over,
t = time of motion.

If the **body starts from rest,** the initial velocity u equals 0, and these equations become:

$$s = \tfrac{1}{2}vt$$
$$s = \tfrac{1}{2}at^2$$
$$2as = v^2$$

Rotation of a Rigid Body

$$\text{velocity at any instant} = \omega = \frac{d\theta}{dt}$$

$$\text{acceleration at any instant} = \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

For motion with **uniform velocity,**

$$\omega = \frac{\theta}{t}$$

For **uniformly accelerated motion,**

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)\,t$$
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$2\,\alpha\theta = \omega^2 - \omega_0^2$$

θ = angular space through which the body rotates,
ω_0 = initial angular velocity,
ω = final angular velocity,
α = angular acceleration,
t = time.

For a **body initially at rest,** the velocity ω_0 is 0, and these equations become

$$\theta = \tfrac{1}{2}\,\omega t$$
$$\theta = \tfrac{1}{2}\,\alpha t^2$$
$$2\,\alpha\theta = \omega^2$$

Falling Bodies

Equations of motion of a **body falling from rest** under the action of gravity:

$$v = gt$$
$$s = \tfrac{1}{2}\,gt^2$$
$$2\,gs = v^2$$

v = velocity after time t,
s = height through which body falls,
g = (approx.) 32.16 feet/sec.^{2} = 981 cm/sec.^{2}
= acceleration of gravity.

The **value of g** for **any latitude** and **any altitude** is

$$g = 32.0894\,(1 + 0.0052375 \sin^2 \theta) \times (1 - 0.0000000957\,E)$$

in which

θ = latitude of place in degrees,
E = elevation above sea-level in feet.

Projectiles

Equations of a body **projected vertically upward** with an initial velocity u (resistance of air not considered):

(1) Velocity at any time $= u - gt$.

(2) Velocity at any height $= \sqrt{u^2 - 2\,gh}$.

(3) Height at any time $= ut - \frac{1}{2}\,gt^2$.

(4) Greatest height $= \dfrac{u^2}{2\,g}$.

(5) Time of flight $= \dfrac{2\,u}{g}$.

Equations of a body **projected** with an **initial** velocity u **at an angle θ° to the horizontal** (resistance of air not considered):

The curve described by the projectile is the parabola whose equation is

$$y = x \tan \theta - \frac{gx^2}{2\,u^2 \cos^2 \theta}$$

where θ is positive when the body is projected above

the horizontal and negative when the body is projected below the horizontal.

Horizontal-component of acceleration $= \dfrac{d^2x}{dt^2} = 0$

Vertical-component of acceleration $= \dfrac{d^2y}{dt^2} = -g$

(1) Velocity at any time $= \sqrt{u^2 - 2\,utg \sin\theta + g^2t^2}$.

(2) Velocity at any height $= \sqrt{u^2 - 2\,gh}$.

(3) Height at any time $= ut \sin\theta - \frac{1}{2}\,gt^2$.

(4) Time of flight $= \dfrac{2\,u \sin\theta}{g}$.

(5) Range $= \dfrac{u^2 \sin(2\,\theta)}{g}$.

If the friction of the air is taken into account, the curve described by the projectile is given by the empirical relation:

$$y = x \tan\theta - \frac{gx^2}{2\cos^2\theta}\left(\frac{1}{u^2} + \frac{kx}{u}\right)$$

$$k = 0.0000000458\,\frac{d^2}{w}$$

where d = diameter of projectile in inches.
w = weight of projectile in pounds.

Angular Measure

A **radian** is the angle subtended at the center of any circle by an arc equal in length to its radius.

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57.296+ \text{ degrees}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} = 0.017453+ \text{ radian}$$

The relation between the central angle of a circle and its subtended arc is given by the formula:

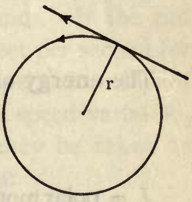
$$l = r\,\theta$$

l = length of arc,
r = radius of circle,
θ = central angle in radians.

Circular Motion

A body moving with **uniform velocity** in a circular path experiences a constant acceleration toward the center of the circle. This acceleration is expended in changing the direction of motion of the body.

The equations of motion of the revolving body are



$$a = \frac{v^2}{r}$$

$$vT = 2\pi r$$

$$a = \frac{4\pi^2 r}{T^2}$$

v = constant velocity of particle in feet per second,
a = constant acceleration toward center in feet per sec.^{2},
r = radius of circular path in feet,
T = time of 1 revolution in seconds,
$\pi^2 = 9.8696+$.

If the body moves with a **variable velocity,** then:

$$\text{tangential acceleration} = \frac{dv}{dt}$$

$$\text{normal acceleration} = \frac{v^2}{r}$$

Centrifugal Force

The centrifugal force of a revolving body, in pounds, is

$$F = \frac{Wv^2}{gr} = \frac{4\pi^2 Wr}{gt^2}$$

or in terms of the number of revolutions, N_1, per minute

$$F = 0.00034\,WrN_1^2$$

W = weight of revolving body in pounds,
v = velocity of body in feet per second,
t = time of 1 revolution in seconds,
r = distance from axis of rotation to the center of gravity of the body, in feet,
g = acceleration of gravity (32.16).

Flywheel

The **energy** of rotation of a flywheel is

$$K.E. = \frac{I\omega^2}{2} = 2\pi^2IN^2$$

I = polar moment of inertia about the axis of rotation,
ω = angular velocity in radians per second,
N = number of revolutions per second.

The **energy** stored in a rim flywheel by a variation in speed is

$$E = \frac{W}{2\,g}(S^2{}_{max} - S^2{}_{min}) \text{ foot-pounds,}$$

W = weight of flywheel in pounds,
S_{max} = maximum rim speed in feet per second,
S_{min} = minimum rim speed in feet per second
g = acceleration of gravity (32.16).

The rim speed in feet per second is $S = 2\pi RN$, where N is the speed in revolutions per second, and R is the radius of the wheel in feet, measured from the center of gravity of the rim section.*

* This value of R is approximately correct. The exact value of R is the radius of gyration of the flywheel.

Hence, the **energy stored** is

$$E = \frac{W}{g} \frac{4\pi^2R^2 (N^2_{max} - N^2_{min})}{2} \text{ foot-pounds}$$

and the **weight** of the flywheel is

$$W = \frac{Eg}{2\pi^2R^2 (N^2_{max} - N^2_{min})}$$

Substitute for E the required stored energy in foot-pounds. Assume some convenient value for R, in feet; then solve for the weight W in pounds. If the rim speed is too high (average about 35 feet per second for cast iron or 150 feet per second for steel), the value of R must be reduced. The ratio of the speed variation, $N_{max} - N_{min}$, to the average speed may be taken as follows for different types of machines:

Hammers	0.20
Punches	0.05
Ordinary machinery	0.03
Textile and paper machinery	0.02
Electric generators	0.005

* This value of R is approximately correct. The exact value of R is the radius of gyration of the flywheel.

Simple Pendulum

The **time of oscillation** in seconds from one extreme position to the other is

$$t = \pi\sqrt{\frac{l}{g}}$$

l = length of pendulum in feet,

g = acceleration of gravity (32.16 approx.).

The **period** of the pendulum is

$$P = 2t = 2\pi\sqrt{\frac{l}{g}}$$

The **seconds-pendulum** makes one oscillation per

second from one extreme position to the other; its length in feet is

$$l = \frac{g}{\pi^2}$$

Work and Energy

For a **uniform** force,

$$F = ma = \frac{W}{g}a$$

$$Ft = mv = \frac{W}{g}v$$

$$Fs = \tfrac{1}{2}mv^2 = \frac{Wv^2}{2g}$$

F = constant applied force in pounds,
a = constant acceleration in feet/sec.^{2},
m = mass of body,
W = weight of body in pounds,
v = velocity acquired after t seconds,
mv = momentum,
s = space passed over in feet,
g = acceleration of gravity (32.16 feet/sec.^{2}).

The **impulse** I of the constant force F during the time t equals the change of momentum,

$$I = Ft = mv - mu$$

where u is the initial velocity and v the final velocity.

If the force is variable, then **impulse** equals

$$I = \int_0^t F\,dt$$

The **work** done by a uniform force is

$$W = Fs = \tfrac{1}{2}mv^2$$

The **work** done by a variable force equals

$$W = \int_0^s F\,ds$$

The **kinetic energy** of a body of mass m, moving with a velocity v, equals $\frac{1}{2}mv^2$.

Direct Central Impact

For the impact of two bodies of the same material, weighing respectively W and W_1 pounds, the velocities after impact are

$$v = \frac{Wu + W_1u_1 - eW_1(u - u_1)}{W + W_1}$$

$$v_1 = \frac{Wu + W_1u_1 + eW(u - u_1)}{W + W_1}$$

u = original velocity of W in feet/second,
v = velocity of W after impact,
u_1 = original velocity of W_1,
v_1 = velocity of W_1 after impact,
e = coefficient of restitution.

Values of e, the coefficient of restitution, for different materials are as follows:

glass on glass..................	$e = 0.94$
ivory on ivory.................	$e = 0.81$
cast iron on cast iron..........	$e = 0.66$
lead on lead...................	$e = 0.2$

The sum of the momenta of two bodies after impact equals the sum of their momenta before impact,

$$\frac{Wv}{g} + \frac{W_1v_1}{g} = \frac{Wu}{g} + \frac{W_1u_1}{g}$$

Two **inelastic** bodies after impact move with a common velocity

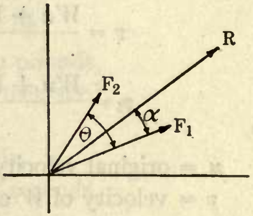
$$v = \frac{W_1v_1 + W_2v_2}{W_1 + W_2}$$

in which

W_1 = weight of first body,
W_2 = weight of second body,
v_1 = original velocity of first body,
v_2 = original velocity of second body.

Composition and Resolution of Forces

The **resultant** of the forces F_1 and F_2 acting at a point is



$$R = \sqrt{F_1^2 + 2\,F_1F_2\cos\theta + F_2^2}$$

in which θ is the angle in degrees between the two forces.

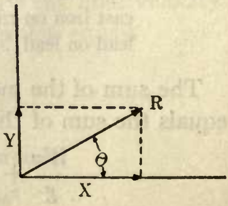
The **direction** of R is determined by the relation

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

in which α is the angle in degrees between F_1 and R.

The **rectangular components** of a force R acting in a given direction are

$$X = R\cos\theta$$
$$Y = R\sin\theta$$



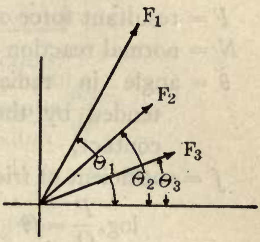
in which X is the horizontal component of R, Y is the normal component of R, and θ is the angle in degrees between R and X.

The **resultant** of several forces acting in different directions at a point is

$$R = \sqrt{X^2 + Y^2}$$

in which

$$X = F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + \cdots,$$
$$Y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 + \cdots,$$

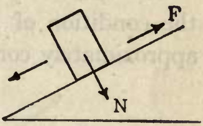


where F_1, F_2, F_3, etc., are the given forces, and θ_1. θ_2, θ_3, etc., are the angles in degrees between the given forces and the horizontal axis.

Friction

F = friction in pounds,
N = normal force in pounds,
f = coefficient of friction.

$$F = fN$$



$$\text{Angle of friction} = \phi = \tan^{-1}\frac{F}{N} = \tan^{-1} f$$

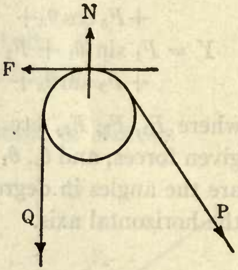
Average values for f, the coefficient of friction, for motion are as follows:

Character of contact	f
Wood on wood	0.25–0.50
Metal on wood	0.50–0.60
Metal on metal, dry	0.15–0.24
Metal on metal, lubricated	0.075
Leather on metal, dry	0.56
Leather on metal, lubricated	0.15

Belt Friction

P and Q are the forces at the ends of the belt, P being the greater force.

F = resultant force of friction,
N = normal reaction of pulley,
θ = angle in radians subtended by the arc of contact,
f = coefficient of friction.



$$\log_e \frac{P}{Q} = f\theta$$

or in common logarithms

$$\log_{10} \frac{P}{Q} = 0.434 f\theta$$

The value of f varies from 0.15 to 0.6 depending on the condition of belt and pulley, but, in general, it is approximately correct to assume $f = 0.3$.

Inclined Plane

Equations of motion of a body sliding down an incline under the action of its own weight.

For a frictionless plane:

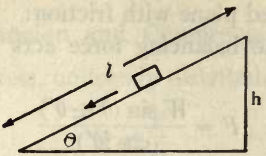
(1) acceleration along plane $= a = \dfrac{d^2s}{dt^2} = g \sin \theta$,

(2) velocity after t seconds $= tg \sin \theta$,

(3) velocity at bottom of plane $= \sqrt{2\,gh}$,

(4) distance traveled in t seconds $= \dfrac{t^2 g \sin \theta}{2}$,

(5) time of sliding down plane $= l \sqrt{\dfrac{2}{gh}}$.



For an inclined plane with friction:

(1) acceleration along plane $= a = \dfrac{d^2s}{dt^2}$

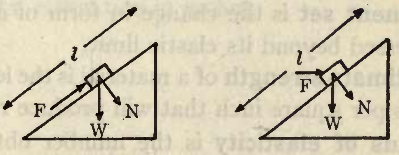
$$= g\,[\sin\theta - f\cos\theta],$$

in which

f = coefficient of friction.

Conditions for the equilibrium of a body resting on an incline:

W = weight of body,
F = applied force,
N = normal pressure on plane,
θ = inclination of plane in degrees,
f = coefficient of friction.



For a frictionless plane:

(1) When the balancing force is applied parallel to the inclined plane,

$$F = W\sin\theta$$
$$N = W\cos\theta$$

(2) When the applied force acts horizontally,

$$F = W\tan\theta,$$
$$N = W\sec\theta.$$

For an inclined plane with friction:

(1) When the balancing force acts parallel to the incline,

$$F = \frac{W \sin(\theta \pm \theta')}{\cos(\theta')}$$

in which

$$\theta' = \tan^{-1} f$$

(2) When the applied force acts horizontally,

$$F = W \tan(\theta \pm \theta')$$

MECHANICS OF MATERIALS

Stress is distributed force; its intensity per unit area is generally expressed in pounds per square inch.

The **elastic limit** of a material is the maximum stress in pounds per square inch that will be followed by a complete recovery of form, after the removal of the stress.

Permanent set is the change in form of a member when stressed beyond its elastic limit.

The **ultimate strength** of a material is the least stress in pounds per square inch that will produce rupture.

Modulus of elasticity is the number obtained by dividing the actual stress in pounds per square inch by the corresponding elongation per inch.

The **factor of safety** is the factor obtained by dividing the ultimate strength by the actual stress in pounds per square inch.

Tension and Compression

For direct stress, uniformly distributed,

$$p = \frac{P}{F}$$

p = stress in pounds per square inch,
P = total load in pounds,
F = cross-sectional area in square inches.

$$E = \frac{p}{\epsilon} \qquad \epsilon = \frac{\lambda}{l}$$

$$E = \frac{\frac{P}{F}}{\frac{\lambda}{l}} = \frac{Pl}{F\lambda}$$

E = modulus of elasticity in tension or compression,
l = length of member in inches,
ϵ = elongation per inch length,
λ = total elongation in inches.

STRENGTH OF MATERIALS

Material	Density	Elastic limit	Ultimate strength			Modulus of elasticity		Factor of safety		
			Tension	Comp.	Shear	Tens. and comp.	Shear	Steady load	Var. load	Shocks
Brick	2	......	......	3,000	1,000	2,000,000	....	15	25	40
Stone	2.6	......	......	6,000	1,500	6,000,000	....	15	25	40
Timber	0.6	3,000	10,000	8,000	......	1,500,000	....	8	10	15
Timber along grain	0.6	......	......	......	500	....	....	....	......	......
Timber across grain	0.6	......	......	......	3,000	....	400,000	....	......	......
Cast iron	7.2	6,000	20,000	90,000	18,000	15,000,000	6,000,000	6	10	20
Wrought iron	7.7	25,000	50,000	50,000	40,000	25,000,000	10,000,000	4	6	10
Structural steel	7.8	35,000	60,000	60,000	50,000	30,000,000	12,000,000	4	6	10
Strong steel	7.8	50,000	100,000	120,000	80,000	30,000,000	12,000,000	5	8	15

Note. — The elastic limit of **6,000** for cast iron holds only for tension; for compression, the elastic limit is **20,000**.

Angular Distortion and Shear

Shearing stress, uniformly distributed equals

$$p_s = \frac{P}{F}$$

P = load,
F = area.

For **torsion:**

$$E_s = \frac{p_s}{\delta}$$

E_s = modulus of elasticity in shear,
δ = angle of distortion in radians.

Note. The modulus of elasticity in shear is $\frac{2}{5}$ as great as in compression or tension.

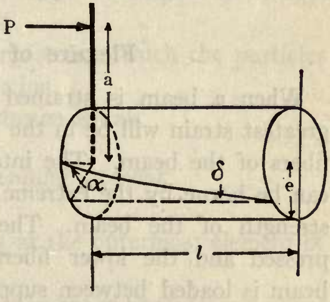
Torsion of Circular Shafts

$$\delta = \frac{e\alpha}{l} \qquad p_s = \frac{e\alpha E_s}{l}$$

$$Pa = \frac{p_s I_p}{e} = \frac{\alpha I_p E_s}{l}$$

$$I_p = \frac{\pi d^4}{32}$$

$$Pa = \frac{\pi p_s d^3}{16}$$



$$\text{Horsepower} = \frac{2\pi PaN}{33{,}000 \times 12}$$

δ = helix angle of distortion in radians,
α = radial angle of distortion in radians,
l = length of shaft in inches,
e = radius of shaft in inches,

p_s = greatest shearing stress in pounds per square inch existing in shaft,
E_s = modulus of elasticity in shear,
I_p = polar moment of inertia of circular section (see table of standard sections),
P = force in pounds producing torsion, that is, the turning force,
a = lever arm of force P in inches,
d = diameter of shaft in inches,
N = revolutions per minute.

In deriving the above formulæ, the torsion is treated as due to a couple of the same turning moment, Pa, as the single force P with lever arm a. This eliminates the consideration of any stresses other than shearing stresses, and, in applying these formulæ to the case of a single driving force, bending stresses and bearing friction are neglected.

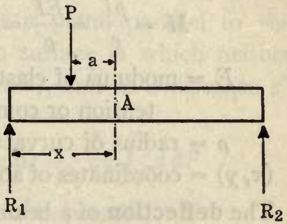
Flexure of Beams

When a beam is strained by a vertical load, the greatest strain will be in the extreme upper and lower fibers of the beam. The intensity of the strain that can be borne by the extreme fibers is the limit of the strength of the beam. The upper fibers are compressed and the lower fibers are stretched when a beam is loaded between supports; the converse holds when it is loaded beyond supports. Somewhere along or near the center of the beam the fibers are neither extended nor compressed; the plane of these fibers is called the **neutral surface.** The line of intersection of the neutral surface with any cross-section of the beam is the **neutral axis** of the section.

If the stresses remain within the elastic limits of the material in both tension and compression, and provided the modulus of elasticity is the same for both kinds of stress, then the **neutral axis** of the section passes through its **center of gravity.**

The **elastic curve** is the curve assumed by a beam under load.

The **bending moment** for any section of a beam is the algebraic sum of the moments of the external or applied forces acting on the beam on one side of the section. Thus, for the beam shown, the bending moment about A is



$$M = R_1x - Pa$$

The bending moment, M, of any section is numerically equal to the **moment of resistance** of the section, which is the resistance which the particles of the beam offer to distortion.

The **moment of resistance** equals

$$\frac{pI}{e} = M = \text{bending moment}$$

p = stress per unit area at the outermost element of the section,

e = distance of extreme element of beam from neutral axis,

I = rectangular moment of inertia of beam section about its horizontal gravity axis.

In designing the proper cross-section for a beam, the maximum bending moment (given for standard cases

under Beam Loadings) is equated to $\frac{pI}{e}$. The term $\frac{I}{e}$, called the section modulus, may be obtained from the table of standard sections of beams. The value of p must not exceed the maximum allowable stress per unit area for the material of the beam. The **maximum allowable stress** equals the ultimate strength divided by the factor of safety.

The **equation of the elastic curve** and its **radius of curvature** may be found from the relations:

$$M = \frac{pI}{e} = \frac{EI}{\rho} = EI\frac{d^2y}{dx^2} \text{ (approx.)}$$

E = modulus of elasticity of material of beam in tension or compression,

ρ = radius of curvature of the elastic curve,

(x, y) = coördinates of any point on the elastic curve.

The **deflection** of a beam at any point is obtained by substituting, in the equation of the elastic curve, the particular value of x in question, and solving for the corresponding value of y, which equals the deflection. The **maximum deflection** occurs at the section for which $\frac{dy}{dx} = 0$.

Shear

The **vertical shear** in a beam is equal to the first derivative of the bending moment in respect to x, thus

$$\text{Vertical shear} = J = \frac{dM}{dx}$$

where M is the bending moment (expressed as a function of x).

The value of the vertical shear for any particular

section is found by substituting the corresponding value of x in the expression for $\frac{dM}{dx}$. The result is the required vertical shear.

The **maximum bending moment** is found by equating $\frac{dM}{dx} = 0$, and then solving for the corresponding value of x. This particular value of x is substituted in the equation of the bending moment, M, and the resulting expression equals the maximum bending moment.

The **horizontal shear** in a plane parallel to the neutral surface (that is, the surface in which neither tension nor compression occurs), and at a distance z'' from it, equals

$$X \text{ (in pounds/sq. inch)} = \frac{J}{y''I}\int_{z''}^{e} z\,dF$$

where J = total vertical shear in pounds,
y'' = width of beam section at z'' in inches,
I = rectangular moment of inertia of entire section about the horizontal gravity axis,
$\int_{z''}^{e} z\,dF$ = area in square inches of that portion of the section above z'' multiplied by the distance in inches of its center of gravity above the neutral axis.

Beam Loadings

M = bending moment,
M_m = maximum bending moment,
y = deflection at any point,
d = maximum deflection,
P = concentrated load,
W = uniformly distributed load.

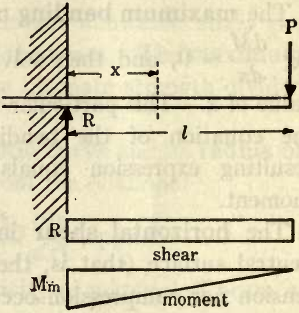
Cantilever Beam with Concentrated Load at the Free End

$$M = P(l - x)$$

$$M_m = Pl$$

$$y = \frac{P}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$$

$$d = \frac{Pl^3}{3\,EI}$$



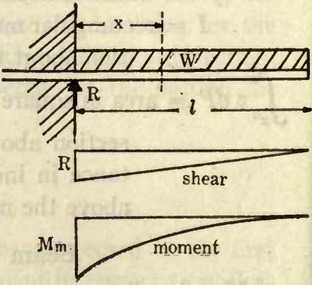
Cantilever Beam with Uniform Load

$$M = \frac{W(l - x)^2}{2\,l}$$

$$M_m = \frac{Wl}{2}$$

$$y = \frac{Wx^2\,[2\,l^2 + (2\,l - x)^2]}{24\,EIl}$$

$$d = \frac{Wl^3}{8\,EI}$$



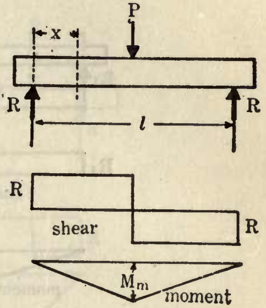
Beam Supported at Both Ends and Loaded with a Concentrated Load at Center

$$M = \frac{P}{2}x$$

$$M_m = \frac{Pl}{4}$$

$$y = \frac{Px\,(3\,l^2 - 4\,x^2)}{48\,EI}$$

$$d = \frac{Pl^3}{48\,EI}$$



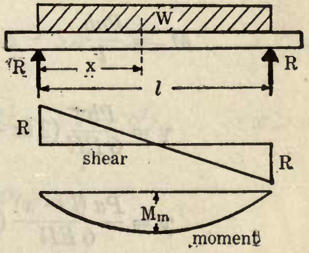
Beam Supported at Both Ends and Uniformly Loaded

$$M = \frac{Wx\,(l - x)}{2\,l}$$

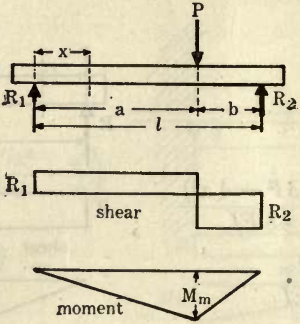
$$M_m = \frac{Wl}{8}$$

$$y = \frac{Wx\,(l^3 - 2\,lx^2 + x^3)}{24\,EIl}$$

$$d = \frac{5\,Wl^3}{384\,EI}$$



Beam Supported at Both Ends and Loaded at Any Point



$$M = \frac{Pbx}{l} \qquad x < a$$

$$M = \frac{Pbx}{l} - P(x - a) \qquad x > a$$

$$M_m = \frac{Pab}{l}$$

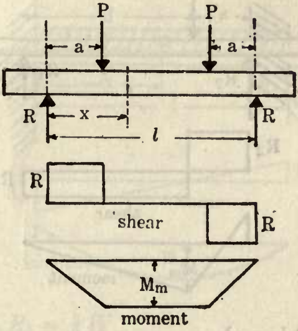
$$y = \frac{Pbx}{6\,EIl}(2\,la - a^2 - x^2) \qquad x < a$$

$$y = \frac{Pa\,(l - x)}{6\,EIl}(2\,lx - x^2 - a^2) \qquad x > a$$

$$d = \frac{Pb}{27\,EIl}\sqrt{3\,(2\,ab + a^2)^3}$$

occurring when $x = \frac{1}{3}\sqrt{3\,(2\,ab + a^2)}$

Beam Supported at Both Ends and Loaded with Two Concentrated Loads at Equal Distances from Each End



$$M = Px \qquad x < a$$

$$M = Pa \qquad x > a$$

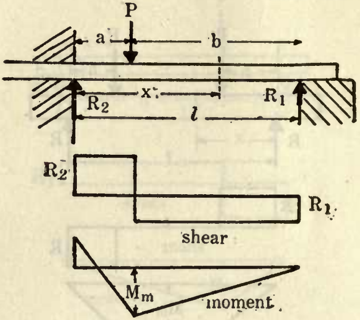
$$M_m = Pa$$

$$y = \frac{Px}{6\,EI}(3\,la - 3\,a^2 - x^2) \qquad x < a$$

$$y = \frac{Pa}{6\,EI}(3\,lx - 3\,x^2 - a^2) \qquad x > a$$

$$d = \frac{Pa}{6\,EI}\left(\frac{3}{4}l^2 - a^2\right)$$

Beam Fixed at One End, Supported at the Other, and with a Concentrated Load at Any Point



$$R_1 = \frac{Pa^2(3\,l - a)}{2\,l^3}$$

$$R_2 = P - R_1$$

$$M = P(a - x) - R_1(l - x) \qquad x < a$$

$$M = R_1(x - l) \qquad x > a$$

$$M_m = R_1(l - a)$$

$$y = \frac{1}{6\,EI}(R_1x^3 - 3\,R_1lx^2 + 3\,Pax^2 - Px^3) \qquad x < a$$

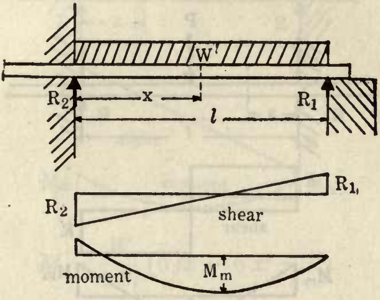
$$y = \frac{1}{6\,EI}(R_1x^3 - 3\,R_1lx^2 + 3\,Pa^2x - Pa^3) \qquad x > a$$

$$d = \frac{Pa^2}{6\,EI}(l - a)\sqrt{\frac{(l - a)}{(3\,l - a)}}$$

occurring when

$$x = l\left(1 - \sqrt{\frac{l - a}{3\,l - a}}\right)$$

Beam Fixed at One End, Supported at the Other and Uniformly Loaded



$$R_1 = \tfrac{3}{8} W$$

$$R_2 = \tfrac{5}{8} W$$

$$M = \frac{W}{8\,l}(l - 4\,x)\,(l - x)$$

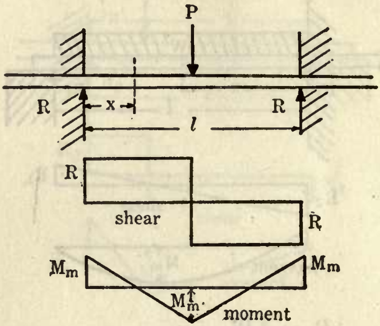
$$M_m = \frac{Wl}{8}$$

$$y = \frac{Wx^2}{48\,EIl}(l - x)\,(3\,l - 2\,x)$$

$$d = 0.0054\,\frac{Wl^3}{EI}$$

occurring when $x = 0.5785\,l$

Beam Fixed at Both Ends and Loaded at the Center



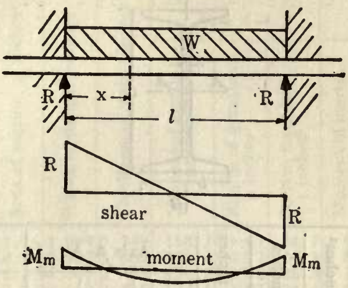
$$M = \frac{P}{8}(4x - l)$$

$$M_m = \frac{Pl}{8}$$

$$y = \frac{Px^2}{48\,EI}(4x - 3\,l)$$

$$d = \frac{Pl^3}{192\,EI}$$

Beam Fixed at Both Ends and Uniformly Loaded



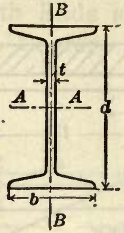
$$M = \frac{W}{12\,l}(6\,lx - 6\,x^2 - l^2)$$

$$M_m = \frac{Wl}{12}$$

$$y = \frac{Wx^2}{24\,EIl}(l - x)^2$$

$$d = \frac{Wl^3}{384\,EI}$$

PROPERTIES OF STANDARD I BEAMS *



Depth of beam, d inches	Weight per foot, w pounds	Area of section, A inches^2	Width of flange, b inches	Thickness of web, t inches	Axis A–A			Axis B–B		
					Moment of inertia, I inches^4	Radius of gyration, k inches	Section modulus, s inches^3	Moment of inertia I inches^4	Radius of gyration, k inches	Section modulus, s inches^3
20	75.0	22.06	6.399	0.649	1268.8	7.58	126.9	30.3	1.17	9.5
	70.0	20.59	6.325	0.575	1219.8	7.70	122.0	29.0	1.19	9.2
	65.0	19.08	6.250	0.500	1169.5	7.83	117.0	27.9	1.21	8.9
18	90.0	26.47	7.245	0.807	1260.4	6.90	140.0	52.0	1.40	14.4
	85.0	25.00	7.163	0.725	1220.7	6.99	135.6	50.0	1.42	14.0
	80.0	23.53	7.082	0.644	1181.0	7.09	131.2	48.1	1.43	13.6
	75.0	22.05	7.000	0.562	1141.3	7.19	126.8	46.2	1.45	13.2
18	70.0	20.59	6.259	0.719	921.2	6.69	102.4	24.6	1.09	7.9
	65.0	19.12	6.177	0.637	881.5	6.79	97.9	23.5	1.11	7.6
	60.0	17.65	6.095	0.555	841.8	6.91	93.5	22.4	1.13	7.3
	55.0	15.93	6.000	0.460	795.6	7.07	88.4	21.2	1.15	7.1
15	75.0	22.06	6.292	0.882	691.2	5.60	92.2	30.7	1.18	9.8
	70.0	20.59	6.194	0.784	663.7	5.68	88.5	29.0	1.19	9.4
	65.0	19.12	6.096	0.686	636.1	5.77	84.8	27.4	1.20	9.0
	60.0	17.67	6.000	0.590	609.0	5.87	81.2	26.0	1.21	8.7
15	55.0	16.18	5.746	0.656	511.0	5.62	68.1	17.1	1.02	5.9
	50.0	14.71	5.648	0.558	483.4	5.73	64.5	16.0	1.04	5.7
	45.0	13.24	5.550	0.460	455.9	5.87	60.8	15.1	1.07	5.4
	42.0	12.48	5.500	0.410	441.8	5.95	58.9	14.6	1.08	5.3
12	55.0	16.18	5.611	0.821	321.0	4.45	53.5	17.5	1.04	6.2
	50.0	14.71	5.489	0.699	303.4	4.54	50.6	16.1	1.05	5.9
	45.0	13.24	5.366	0.576	285.7	4.65	47.6	14.9	1.06	5.6
	40.0	11.84	5.250	0.460	269.0	4.77	44.8	13.8	1.08	5.3
12	35.0	10.29	5.086	0.436	228.3	4.71	38.0	10.1	0.99	4.0
	31.5	9.26	5.000	0.350	215.8	4.83	36.0	9.5	1.01	3.8

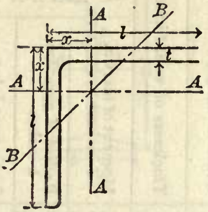
* Manufactured by the Carnegie Steel Company, Pittsburg, Pa.

PROPERTIES OF STANDARD I BEAMS * (*Continued*)

Depth of beam, d inches	Weight per foot, w pounds	Area of section, A inches²	Width of flange, b inches	Thickness of web, t inches	Axis A-A			Axis B-B		
					Moment of inertia, I inches⁴	Radius of gyration, k inches	Section modulus, s inches³	Moment of inertia, I inches⁴	Radius of gyration, k inches	Section modulus, s inches³
10	40.0	11.76	5.099	0.749	158.7	3.67	31.7	9.5	0.90	3.7
	35.0	10.29	4.952	0.602	146.4	3.77	29.3	8.5	0.91	3.4
	30.0	8.82	4.805	0.455	134.2	3.90	26.8	7.7	0.93	3.2
	25.0	7.37	4.660	0.310	122.1	4.07	24.4	6.9	0.97	3.0
9	35.0	10.29	4.772	0.732	111.8	3.29	24.8	7.3	0.84	3.1
	30.0	8.82	4.609	0.569	101.9	3.40	22.6	6.4	0.85	2.8
	25.0	7.35	4.446	0.406	91.9	3.54	20.4	5.7	0.88	2.5
	21.0	6.31	4.330	0.290	84.9	3.67	18.9	5.2	0.90	2.4
8	25.5	7.50	4.271	0.541	68.4	3.02	17.1	4.8	0.80	2.2
	23.0	6.76	4.179	0.449	64.5	3.09	16.1	4.4	0.81	2.1
	20.5	6.03	4.087	0.357	60.6	3.17	15.2	4.1	0.82	2.0
	18.0	5.33	4.000	0.270	56.9	3.27	14.2	3.8	0.84	1.9
7	20.0	5.88	3.868	0.458	42.2	2.68	12.1	3.2	0.74	1.7
	17.5	5.15	3.763	0.353	39.2	2.76	11.2	2.9	0.76	1.6
	15.0	4.42	3.660	0.250	36.2	2.86	10.4	2.7	0.78	1.5
6	17.25	5.07	3.575	0.475	26.2	2.27	8.7	2.4	0.68	1.3
	14.75	4.34	3.452	0.352	24.0	2.35	8.0	2.1	0.69	1.2
	12.25	3.61	3.330	0.230	21.8	2.46	7.3	1.9	0.72	1.1
5	14.75	4.34	3.294	0.504	15.2	1.87	6.1	1.7	0.63	1.0
	12.25	3.60	3.147	0.357	13.6	1.94	5.5	1.5	0.63	0.92
	9.75	2.87	3.000	0.210	12.1	2.05	4.8	1.2	0.65	0.82

* Manufactured by the Carnegie Steel Company, Pittsburg, Pa.

PROPERTIES OF STANDARD ANGLES WITH EQUAL LEGS *



				Axis A-A				Axis B-B
Size, l inches	Thickness, t inches	Weight per foot, w pounds	Area of section, A inches²	Distance from back of angle to center of gravity, x ins.	Moment of inertia, I inches⁴	Radius of gyration, k inches	Section modulus, s inches³	Minimum radius of gyration, k ins.
6×6	1	37.4	11.00	1.86	35.5	1.80	8.6	1.16
	$\frac{15}{16}$	35.3	10.37	1.84	33.7	1.80	8.1	1.16
	$\frac{7}{8}$	33.1	9.73	1.82	31.9	1.81	7.6	1.17
	$\frac{13}{16}$	31.0	9.09	1.80	30.1	1.82	7.2	1.17
	$\frac{3}{4}$	28.7	8.44	1.78	28.2	1.83	6.7	1.17
	$\frac{11}{16}$	26.5	7.78	1.75	26.2	1.83	6.2	1.17
	$\frac{5}{8}$	24.2	7.11	1.73	24.2	1.84	5.7	1.17
	$\frac{9}{16}$	21.9	6.43	1.71	22.1	1.85	5.1	1.18
	$\frac{1}{2}$	19.6	5.75	1.68	19.9	1.86	4.6	1.18
	$\frac{7}{16}$	17.2	5.06	1.66	17.7	1.87	4.1	1.19
	$\frac{3}{8}$	14.9	4.36	1.64	15.4	1.88	3.5	1.19
4×4	$\frac{3}{4}$	18.5	5.44	1.27	7.7	1.19	2.8	0.77
	$\frac{11}{16}$	17.1	5.03	1.25	7.2	1.19	2.6	0.77
	$\frac{5}{8}$	15.7	4.61	1.23	6.7	1.20	2.4	0.77
	$\frac{9}{16}$	14.3	4.18	1.21	6.1	1.21	2.2	0.78
	$\frac{1}{2}$	12.8	3.75	1.18	5.6	1.22	2.0	0.78
	$\frac{7}{16}$	11.3	3.31	1.16	5.0	1.23	1.8	0.78
	$\frac{3}{8}$	9.8	2.86	1.14	4.4	1.23	1.5	0.79
	$\frac{5}{16}$	8.2	2.40	1.12	3.7	1.24	1.3	0.79
3½×3½	$\frac{5}{8}$	13.6	3.98	1.10	4.3	1.04	1.8	0.68
	$\frac{9}{16}$	12.4	3.62	1.08	4.0	1.05	1.6	0.68
	$\frac{1}{2}$	11.1	3.25	1.06	3.6	1.06	1.5	0.68
	$\frac{7}{16}$	9.8	2.87	1.04	3.3	1.07	1.3	0.68
	$\frac{3}{8}$	8.5	2.48	1.01	2.9	1.07	1.2	0.69
	$\frac{5}{16}$	7.2	2.09	0.99	2.5	1.08	0.98	0.69

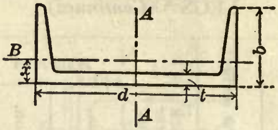
* Manufactured by the Carnegie Steel Company, Pittsburg, Pa.

PROPERTIES OF STANDARD ANGLES WITH EQUAL LEGS * (*Continued*)

Size, l inches	Thickness, t inches	Weight per foot, w pounds	Area of section, A inches²	Axis A-A: Distance from back of angle to center of gravity, x ins.	Axis A-A: Moment of inertia, I inches⁴	Axis A-A: Radius of gyration, k inches	Axis A-A: Section modulus, s inches³	Axis B-B: Minimum radius of gyration, k ins.
3×3	$\frac{1}{2}$	9.4	2.75	0.93	2.2	0.90	1.1	0.58
	$\frac{7}{16}$	8.3	2.43	0.91	2.0	0.91	0.95	0.58
	$\frac{3}{8}$	7.2	2.11	0.89	1.8	0.91	0.83	0.58
	$\frac{5}{16}$	6.1	1.78	0.87	1.5	0.92	0.71	0.59
	$\frac{1}{4}$	4.9	1.44	0.84	1.2	0.93	0.58	0.59
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{3}{8}$	5.9	1.73	0.76	0.98	0.75	0.57	0.48
	$\frac{5}{16}$	5.0	1.47	0.74	0.85	0.76	0.48	0.49
	$\frac{1}{4}$	4.1	1.19	0.72	0.70	0.77	0.39	0.49
	$\frac{3}{16}$	3.07	0.90	0.69	0.55	0.78	0.30	0.49
2×2	$\frac{3}{8}$	4.7	1.36	0.64	0.48	0.59	0.35	0.39
	$\frac{5}{16}$	3.92	1.15	0.61	0.42	0.60	0.30	0.39
	$\frac{1}{4}$	3.19	0.94	0.59	0.35	0.61	0.25	0.39
	$\frac{3}{16}$	2.44	0.71	0.57	0.28	0.62	0.19	0.40

* Manufactured by the Carnegie Steel Company, Pittsburg, Pa.

PROPERTIES OF STANDARD CHANNELS *



Depth of channel, d inches	Weight per foot, w pounds	Area of section, A inches²	Width of flange, b inches	Thickness of web, t inches	Axis A-A: Moment of inertia, I inches⁴	Axis A-A: Radius of gyration, k inches	Axis A-A: Section modulus, s inches³	Axis B-B: Distance from back of web to center of gravity, x inches	Axis B-B: Moment of inertia, I inches⁴	Axis B-B: Radius of gyration, k inches	Axis B-B: Section modulus, s inches³
	40	11.76	3.418	0.758	196.9	4.09	32.8	0.72	6.6	0.75	2.5
	35	10.29	3.296	0.636	179.3	4.17	29.9	0.69	5.9	0.76	2.3
12	30	8.82	3.173	0.513	161.7	4.28	26.9	0.68	5.2	0.77	2.1
	25	7.35	3.050	0.390	144.0	4.43	24.0	0.68	4.5	0.79	1.9
	20.5	6.03	2.940	0.280	128.1	4.61	21.4	0.70	3.9	0.81	1.7
	35	10.29	3.183	0.823	115.5	3.35	23.1	0.70	4.7	0.67	1.9
	30	8.82	3.036	0.676	103.2	3.42	20.7	0.65	4.0	0.67	1.7
10	25	7.35	2.889	0.529	91.0	3.52	18.2	0.62	3.4	0.68	1.5
	20	5.88	2.742	0.382	78.7	3.66	15.7	0.61	2.9	0.70	1.3
	15	4.46	2.600	0.240	66.9	3.87	13.4	0.64	2.3	0.72	1.2
	25	7.35	2.815	0.615	70.7	3.10	15.7	0.62	3.0	0.64	1.4
9	20	5.88	2.652	0.452	60.8	3.21	13.5	0.59	2.5	0.65	1.2
	15	4.41	2.488	0.288	50.9	3.40	11.3	0.59	2.0	0.67	1.0
	13.25	3.89	2.430	0.230	47.3	3.49	10.5	0.61	1.8	0.67	0.97
	21.25	6.25	2.622	0.582	47.8	2.77	11.9	0.59	2.3	0.60	1.1
	18.75	5.51	2.530	0.490	43.8	2.82	11.0	0.57	2.0	0.60	1.0
8	16.25	4.78	2.439	0.399	39.9	2.89	10.0	0.56	1.8	0.61	0.95
	13.75	4.04	2.347	0.307	36.0	2.98	9.0	0.56	1.6	0.62	0.87
	11.25	3.35	2.260	0.220	32.3	3.11	8.1	0.58	1.3	0.63	0.79
	19.75	5.81	2.513	0.633	33.2	2.39	9.5	0.58	1.9	0.56	0.96
	17.25	5.07	2.408	0.528	30.2	2.44	8.6	0.56	1.6	0.57	0.87
7	14.75	4.34	2.303	0.423	27.2	2.50	7.8	0.54	1.4	0.57	0.79
	12.25	3.60	2.198	0.318	24.2	2.59	6.9	0.53	1.2	0.58	0.71
	9.75	2.85	2.090	0.210	21.1	2.72	6.0	0.55	0.98	0.59	0.63
	15.5	4.56	2.283	0.563	19.5	2.07	6.5	0.55	1.3	0.53	0.74
6	13.0	3.82	2.160	0.440	17.3	2.13	5.8	0.52	1.1	0.53	0.65
	10.5	3.09	2.038	0.318	15.1	2.21	5.0	0.50	0.88	0.53	0.57
	8.0	2.38	1.920	0.200	13.0	2.34	4.3	0.52	0.70	0.54	0.50
	11.5	3.38	2.037	0.477	10.4	1.75	4.2	0.51	0.82	0.49	0.54
5	9.0	2.65	1.890	0.330	8.9	1.83	3.6	0.48	0.64	0.49	0.45
	6.5	1.95	1.750	0.190	7.4	1.95	3.0	0.49	0.48	0.50	0.38
	7.25	2.13	1.725	0.325	4.6	1.46	2.3	0.46	0.44	0.46	0.35
4	6.25	1.84	1.652	0.252	4.2	1.51	2.1	0.46	0.38	0.45	0.32
	5.25	1.55	1.580	0.180	3.8	1.56	1.9	0.46	0.32	0.45	0.29

* Manufactured by the Carnegie Steel Company, Pittsburg, Pa.

COLUMNS

Note. The breaking load in Euler's and in Gordon's formula, and the safe load in Ritter's formula are in pounds. In all of the formulæ for columns, the length, l, and radius of gyration, k, must be expressed in the same units (generally inches).

Euler's Formula

(1) Column with round ends,

$$\text{breaking load} = EI\frac{\pi^2}{l^2} = \pi^2 EF\left(\frac{k^2}{l^2}\right)$$

(2) Column with flat ends,

$$\text{breaking load} = 4\,EI\frac{\pi^2}{l^2} = 4\,\pi^2 EF\left(\frac{k^2}{l^2}\right).$$

(3) Pin-and-square column (column with one end round and the other flat),

$$\text{breaking load} = \frac{9}{4}EI\frac{\pi^2}{l^2} = \frac{9}{4}\pi^2 EF\left(\frac{k^2}{l^2}\right)$$

in which

E = modulus of elasticity of material of column in tension or compression,

I = rectangular moment of inertia of cross-section about neutral axis,

l = length of column,

F = area of cross-section in sq. inches,

k = least radius of gyration of section.

Gordon's or Rankine's Formula

(1) Column with flat ends,

$$\text{breaking load} = \frac{FC}{1 + \beta\left(\frac{l}{k}\right)^2}$$

(2) Column with rounded ends,

$$\text{breaking load} = \frac{FC}{1 + 4\beta\left(\frac{l}{k}\right)^2}$$

(3) Pin-and-square column,

$$\text{breaking load} = \frac{FC}{1 + 1.78\beta\left(\frac{l}{k}\right)^2}$$

in which

F = area of cross-section in square inches,
C = ultimate compressive strength of material of column in pounds per square inch,
l = length of column,
k = least radius of gyration of section,
β = empirical constant.

Values of β and of C, in Gordon's formula, are as follows for different materials:

Material	Hard steel	Medium steel	Soft steel	Wrought iron	Cast iron	Timber
C (lbs./sq. in.)	70,000	50,000	45,000	36,000	70,000	7200
β	$\frac{1}{25,000}$	$\frac{1}{36,000}$	$\frac{1}{36,000}$	$\frac{1}{36,000}$	$\frac{1}{6400}$	$\frac{1}{3000}$

Ritter's Formula

(1) Column with flat ends,

$$\text{safe load} = \frac{FC}{1 + \frac{C'}{4\pi^2 E}\left(\frac{l}{k}\right)^2}$$

(2) Column with rounded ends,

$$\text{safe load} = \frac{FC}{1 + \frac{C'}{\pi^2 E}\left(\frac{l}{k}\right)^2}$$

(3) Pin-and-square column,

$$\text{safe load} = \frac{FC}{1 + \frac{1.78\,C'}{4\pi^2 E}\left(\frac{l}{k}\right)^2}$$

in which

F = area of cross-section in square inches,
C = maximum safe compressive stress of material of column in pounds per square inch,
C' = compressive stress at elastic limit in pounds per square inch,
E = modulus of elasticity for tension or compression,
l = length of column,
k = least radius of gyration.

J. B. Johnson's Formula

Breaking load in pounds; cross-section in square inches.

For **mild steel:**

(1) Pin-ends,

$$\text{breaking load} = \left[42{,}000 - 0.97\left(\frac{l}{k}\right)^2\right]F, \quad \left(\frac{l}{k}\right) \text{not} > 150$$

(2) Flat ends,

$$\text{breaking load} = \left[42{,}000 - 0.62\left(\frac{l}{k}\right)^2\right]F$$

$$\left(\frac{l}{k}\right) \text{ not} > 190$$

For wrought iron:

(1) Pin-ends,

$$\text{breaking load} = \left[34{,}000 - 0.67\left(\frac{l}{k}\right)^2\right]F$$

$$\left(\frac{l}{k}\right) \text{ not} > 170$$

(2) Flat ends,

$$\text{breaking load} = \left[34{,}000 - 0.43\left(\frac{l}{k}\right)^2\right]F$$

$$\left(\frac{l}{k}\right) \text{ not} > 210$$

Notation same as in Ritter's formula.

Straight-line Formula

Breaking load in pounds; cross-section in square inches.

For mild steel:

(1) Hinged ends,

$$\text{breaking load} = \left[52{,}000 - 220\left(\frac{l}{k}\right)\right]F$$

(2) Flat ends,

$$\text{breaking load} = \left[52{,}000 - 179\left(\frac{l}{k}\right)\right]F$$

For wrought iron:

(1) Hinged ends,

$$\text{breaking load} = \left[42{,}000 - 157\left(\frac{l}{k}\right)\right]F$$

(2) Flat ends,

$$\text{breaking load} = \left[42{,}000 - 128\left(\frac{l}{k}\right)\right]F$$

Notation same as in Ritter's formula.

Wooden Columns

The breaking load in pounds for solid wooden columns with square ends is

$$P = \frac{(700 + 15\,m)\,FC}{700 + 15\,m + m^2}$$

F = cross-section in square inches,
m = ratio of the length, l, of the column to the least dimension d, of the cross-section $\left(\text{that is, } m = \frac{l}{d}\right)$,
C = ultimate compressive strength of material of column in pounds per square inch.

Values of C, the ultimate compressive strength, for different kinds of timber are as follows:

White oak and Georgia yellow pine.....	5000 lb./sq. in.
Douglas fir and short-leaf yellow pine...	4500 lb./sq. in.
Red pine, spruce, hemlock, cypress, chestnut, California redwood, and California spruce....................	4000 lb./sq. in.
White pine and cedar.................	3500 lb./sq. in.

The **proper factor of safety** for **yellow pine** varies from 3.5 to 5, according to the amount of moisture present in the timber, being greater for larger amounts of moisture. For all other timbers, the proper factor of safety varies from 4 to 5.

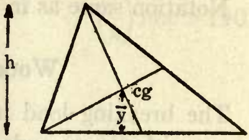
CENTERS OF GRAVITY

Plane Figures

Triangle

The C.G. is on a median line of the triangle, two-thirds of its length from the vertex,

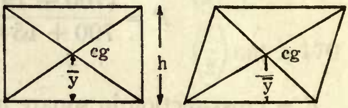
$$\bar{y} = \frac{h}{3}$$



Parallelogram

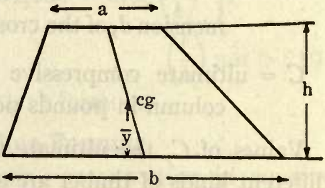
The C.G. is at the intersection of the diagonals,

$$\bar{y} = \frac{h}{2}$$



Trapezoid

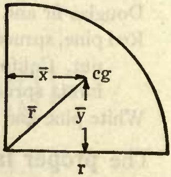
$$\bar{y} = \frac{h\,(2\,a + b)}{3\,(a + b)}$$



Quadrant of Circle

$$\bar{x} = \frac{4\,r}{3\,\pi} = \bar{y}$$

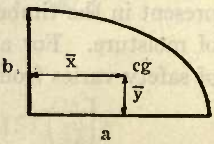
$$\bar{r} = \frac{4\,r\,\sqrt{2}}{3\,\pi}$$



Quadrant of Ellipse

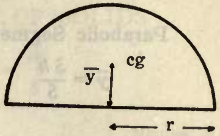
$$\bar{x} = \frac{4\,a}{3\,\pi}$$

$$\bar{y} = \frac{4\,b}{3\,\pi}$$



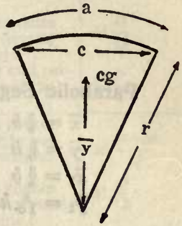
Semicircle

$$\bar{y} = \frac{4r}{3\pi}$$



Circular Sector

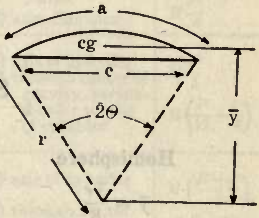
$$\bar{y} = \frac{2rc}{3a}$$



Circular Segment

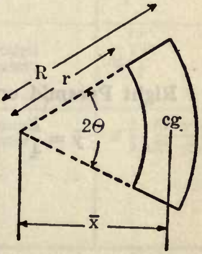
$$\bar{y} = \frac{4}{3}\frac{r\sin^3\theta}{2\theta - \sin(2\theta)}$$

θ is in radians



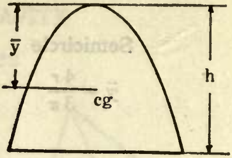
Sector of a Circular Ring

$$\bar{x} = \frac{2}{3}\frac{R^3 - r^3}{R^2 - r^2}\frac{\sin\theta}{\theta}$$



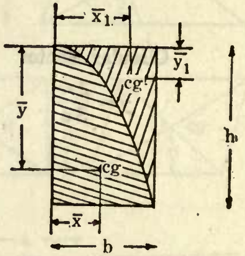
Parabolic Segment

$$\bar{y} = \frac{3h}{5}$$



Parabolic Segment

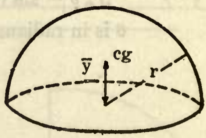
$$\bar{x} = \tfrac{3}{8}b$$
$$\bar{y} = \tfrac{3}{5}h$$
$$\bar{x}_1 = \tfrac{3}{4}b$$
$$\bar{y}_1 = \tfrac{3}{10}h$$



Solids

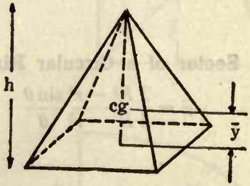
Hemisphere

$$\bar{y} = \frac{3r}{8}$$



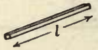
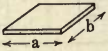
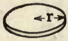
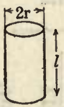
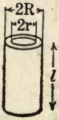
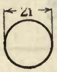
Right Pyramid or Cone

$$\bar{y} = \frac{h}{4}$$

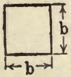
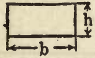
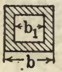
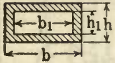


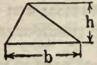
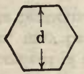
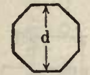
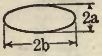
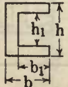
MOMENT OF INERTIA OF SOLIDS

$$M = \text{mass of body} = \frac{W}{g}$$

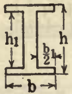
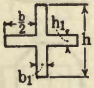
Shape of figure	Description	Axis of rotation	Moment of inertia
	uniform thin rod	(1) through center perpendicular to length (2) through end perpendicular to length	$M\frac{l^2}{12}$ $M\frac{l^2}{3}$
	thin rectangular plate	(1) through center of gravity perpendicular to plate (2) through center of gravity parallel to side b	$M\frac{a^2+b^2}{12}$ $M\frac{a^2}{12}$
	thin circular plate	(1) through center perpendicular to plane (2) any diameter	$M\frac{r^2}{2}$ $M\frac{r^2}{4}$
	solid cylinder, radius, r	(1) axis of cylinder (2) through center of gravity, perpendicular to axis of cylinder	$M\frac{r^2}{2}$ $M\left(\frac{l^2}{12}+\frac{r^2}{4}\right)$
	hollow cylinder, R=outer radius, r=inner radius	(1) axis of cylinder (2) through center of gravity perpendicular to axis	$M\left(\frac{R^2+r^2}{2}\right)$ $M\left(\frac{l^2}{12}+\frac{R^2+r^2}{4}\right)$
	solid sphere, r=radius	through center	$M\frac{2\,r^2}{5}$
	hollow sphere, R=external radius r=internal radius	through center	$M\frac{2}{5}\left(\frac{R^5-r^5}{R^3-r^3}\right)$

PROPERTIES OF STANDARD SECTIONS

Shape of section	Area of section	Rectangular moment of inertia about horizontal gravity axis I	Square of radius of gyration $k^2 = \frac{I}{A}$	Section modulus $s = \frac{I}{e}$	Polar moment of inertia, I_p about center of gravity
	b^2	$\frac{b^4}{12}$	$\frac{b^2}{12}$	$\frac{b^3}{6}$	$\frac{b^4}{6}$
	bh	$\frac{bh^3}{12}$	$\frac{h^2}{12}$	$\frac{bh^2}{6}$	$\frac{bh}{12}(b^2+h^2)$
	$b^2-b_1^2$	$\frac{b^4-b_1^4}{12}$	$\frac{b^2+b_1^2}{12}$	$\frac{b^4-b_1^4}{6\,b}$	$\frac{b^4-b_1^4}{\quad}$
	$bh-b_1h_1$	$\frac{bh^3-b_1h_1^3}{12}$	$\frac{bh^3-b_1h_1^3}{12\,(bh-b_1h_1)}$	$\frac{bh^3-b_1h_1^3}{6\,h}$	$\frac{bh\,(h^2+b^2)}{12} - \frac{b_1h_1\,(h_1^2+b_1^2)}{12}$
	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	$\frac{d^2}{16}$	$\frac{\pi d^3}{32}$	$\frac{\pi d^4}{32}$

	$\pi\,\dfrac{(d^2-d_1^2)}{4}$	$\pi\,\dfrac{(d^4-d_1^4)}{64}$	$\dfrac{d^2+d_1^2}{16}$	$\pi\,\dfrac{(d^4-d_1^4)}{32\,d}$	$\pi\,\dfrac{(d^4-d_1^4)}{32}$
	$\dfrac{bh}{2}$	$\dfrac{bh^3}{36}$	$\dfrac{h^2}{18}$	$\dfrac{bh^2}{24}$	
	$0.866\,d^2$	$0.06\,d^4$	$0.0697\,d^2$	$0.12\,d^3$	
	$0.828\,d^2$	$0.055\,d^4$	$0.066\,d^2$	$0.109\,d^3$	
	πab	$\dfrac{\pi ba^3}{4}$	$\dfrac{a^2}{4}$	$\pi\,\dfrac{ba^2}{4}$	$\dfrac{\pi}{4}\,(a^3b+b^3a)$
	$bh-b_1h_1$	$\dfrac{bh^3-b_1h_1^3}{12}$	$\dfrac{1}{12}\left(\dfrac{bh^3-b_1h_1^3}{bh-b_1h_1}\right)$	$\dfrac{bh^3-b_1h_1^3}{6\,h}$	

PROPERTIES OF STANDARD SECTIONS — *Continued*

	$bh-b_1h_1$	$\dfrac{bh^3-b_1h_1^3}{12}$	$\dfrac{1}{12}\left(\dfrac{bh^3-b_1h_1^3}{bh-b_1h_1}\right)$	$\dfrac{bh^3-b_1h_1^3}{6\,h}$	
	b_1h+bh_1	$\dfrac{b_1h^3+bh_1^3}{12}$	$\dfrac{1}{12}\left(\dfrac{b_1h^3+bh_1^3}{b_1h+bh_1}\right)$	$\dfrac{b_1h^3+bh_1^3}{6\,h}$	

The moment of inertia of such sections as T-beams and angle-bars, the center of gravity of which cannot be determined by inspection, may be obtained as follows: First, find the position of the horizontal gravity axis by the method given on page **74**, for Composite Sections. Then divide the section into its component rectangles, with their bases along the gravity axis. The moment of inertia of each rectangle about its base is calculated by the formula $I = \dfrac{bh^3}{3}$ where b is the base of the rectangle and h its altitude.

The total moment of inertia of the section about its gravity axis is the sum of the moments of inertia of the component rectangles. If there is a rectangular space in the figure, the corresponding moment of inertia is subtracted from that of the solid section.

HYDRAULICS

Head and Pressure

The difference in level of water between two points is called the **head.**

The **pressure** in pounds per square inch at any depth is

$$p = 0.433\,h$$

in which

h = head or depth in feet of water,
0.433 = weight of a column of water 1 foot high and 1 inch in cross-section.

The pressure on a **submerged surface** is always normal to the surface, and equals

$$P \text{ (in pounds)} = 0.433\,hF$$

h = depth of water in feet from the surface of the liquid to the center of gravity of the submerged surface,
F = area of submerged surface in square inches.

Center of Pressure

The **center of pressure** of a submerged surface is the point of application of the resultant of all the fluid pressures on such surface.

The distance of the center of pressure of a **vertical submerged plate** below the liquid surface is

$$d \text{ (in feet)} = \frac{I_s}{F\bar{z}}$$

F = area of plate in square feet,

$\bar{z}$ = distance in feet from the liquid surface to the center of gravity of the plate,

I_s = rectangular moment of inertia of plate about the line of intersection of its plane with the surface of the liquid.

The distance of the center of pressure of a **submerged plate inclined** at an angle θ with the surface is

$$d \text{ (in feet)} = \frac{I_g \sin^2 \theta}{F\bar{z}} + \bar{z}$$

$\bar{z}$ = distance from the liquid surface to the center of gravity of the plate in feet,

F = area of plate in square feet,

I_g = moment of inertia of plate about its gravity axis parallel to the liquid surface.

Flow through Apertures

Due to friction, the velocity of discharge through an aperture in a thin plate or plank is reduced about 3 per cent below its theoretical value. Further, on leaving the orifice, the jet contracts to approximately 64 per cent of the area of the aperture.

The **theoretical** velocity of discharge through a small aperture, in feet per second, is

$$v = \sqrt{2\,gh}$$

g = acceleration of gravity = 32.16,

h = head in feet.

The **actual** velocity of discharge in feet per second is

$$v = \phi\sqrt{2\,gh} = 0.97\sqrt{2\,gh}$$

ϕ = coefficient of velocity.

The **discharge** through the aperture in cubic feet per second is

$$Q = CF\phi\sqrt{2\,gh} = 0.62\,F\sqrt{2\,gh}$$

C = 0.64 (approx.) = coefficient of contraction,
F = area of aperture in square feet.

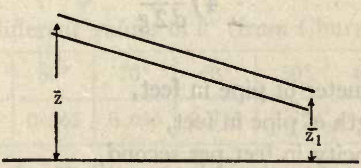
FLOW OF WATER IN PIPES

Bernoulli's Theorem

A general method for calculating the flow of water in pipes is given by Bernoulli's theorem:

$$\frac{v^2}{2\,g} + \frac{p}{\gamma} + \bar{z} = \frac{v_1^2}{2\,g} + \frac{p_1}{\gamma} + \bar{z}_1 + k$$

that is, the sum of the velocity head $\frac{v^2}{2\,g}$, the pressure head $\frac{p}{\gamma}$ and the potential head $\bar{z}$ at any given section of flow is equal to the sum of the corresponding heads at any other section, plus the various losses between the two sections considered.



v = velocity in feet per second at first section,
v_1 = velocity at second section,
p = pressure in pounds per square inch at first section,
p_1 = pressure at second section,

$\bar{z}$ = potential head at first section in feet, that is, the distance of the center of the section above a chosen horizontal reference plane,

$\bar{z}_1$ = potential head at second section,

g = 32.16 (approx.),

γ = weight in pounds of a column of water 1 foot high and 1 square inch in cross-section = 0.433,

k = various losses in feet of head between the two sections of pipe considered.

Losses in Pipes

The following formulæ for losses in pipes enable us to find the value of the term k appearing in Bernoulli's theorem. If several losses occur in a section of pipe, the total loss, k, is the sum of the separate losses.

Loss Due to Friction

The loss of head in feet due to friction in a section of pipe is

$$4f\frac{l}{d}\frac{v^2}{2g}$$

where

d = diameter of pipe in feet,

l = length of pipe in feet,

v = velocity in feet per second,

f = coefficient of friction, depending on the velocity, and on the size of pipe.

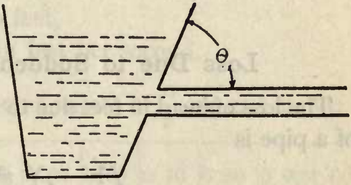
Values of f, the coefficient of friction, for water in clean iron pipes are as follows (condensed from I. P. Church's "Mechanics of Engineering"):

Velocity in feet per second	Diam. =½ in. = 0.0417 ft.	Diam. =1 in. = 0.0834 ft.	Diam. =2 in. =0.1667 ft.	Diam. =4 in. =0.333 ft.	Diam. =8 in. =0.667 ft.	Diam. =12 in. =1.00 ft.	Diam. =16 in. =1.333 ft.	Diam. =20 in. =1.667 ft.
0.1	0.0150	0.0119	0.00870	0.00763	0.00704	0.00669	0.00623	......
0.3	0.0137	0.0113	0.00850	0.00750	0.00693	0.00657	0.00614	0.00578
0.6	0.0124	0.0104	0.00822	0.00732	0.00677	0.00642	0.00603	0.00567
1.0	0.0110	0.00950	0.00790	0.00712	0.00659	0.00624	0.00588	0.00555
2.0	0.00862	0.00810	0.00731	0.00678	0.00624	0.00593	0.00559	0.00529
3.0	0.00753	0.00734	0.00692	0.00650	0.00600	0.00570	0.00538	0.00509
6.0	0.00689	0.00670	0.00640	0.00605	0.00562	0.00534	0.00507	0.00482
12.0	0.00630	0.00614	0.00590	0.00560	0.00522	0.00500	0.00478	0.00457
20.0	0.00615	0.00598	0.00579	0.00549	0.00508	0.00485	......	......

Loss at Entrance

The loss of head in feet due to entrance from a reservoir into a pipe is equal to

$$\left(\frac{1}{\phi^2} - 1\right)\frac{v^2}{2g} = L_e\frac{v^2}{2g}$$



in which ϕ is the coefficient of friction and is dependent on the angle $\theta°$ which the pipe makes with the inner surface of the reservoir.

Values of $L_e\left(= \frac{1}{\phi^2} - 1\right)$ in the above formula are as follows for different values of $\theta°$ (from Church):

$\theta°$	90°	80°	70°	60°	50°	40°	30°
L_e	0.505	0.565	0.635	0.713	0.794	0.870	0.987

Thus, when the discharge is through a pipe normal to the inner surface of the reservoir, then $\theta°$ equals 90° and L_e is, therefore, 0.505, the loss at entrance then being

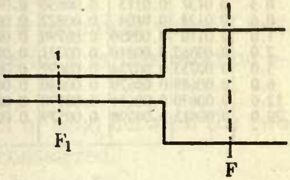
$$0.505\frac{v^2}{2g}$$

where v = velocity of flow in pipe in feet per second.

Loss Due to Sudden Enlargement

The loss of head in feet due to the sudden enlargement of a pipe is

$$\left(\frac{F}{F_1} - 1\right)^2 \frac{v^2}{2g}$$



F_1 = cross-section area of the smaller pipe in square feet,

F = area of enlarged section in square feet,

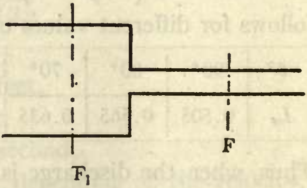
v = velocity in feet per second in the enlarged section.

Loss Due to Sudden Contraction

The loss of head in feet due to the sudden contraction of a pipe is

$$\left(\frac{1}{C} - 1\right)^2 \frac{v^2}{2g}$$

in which



v = velocity in feet per second in contracted section,

C = coefficient of contraction, the value of which depends on the ratio, $\frac{F}{F_1}$, of the small section to the large section.

Values of C, the coefficient of contraction, for

different values of $\frac{F}{F_1}$ are given in the following table (from Church):

$\frac{F}{F_1}$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0
C	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.0

Loss Due to Bends

The loss of head in feet due to a bend in a circular pipe is

$$\left[0.131 + 1.847 \left(\frac{a}{r}\right)^{\frac{7}{2}}\right] \frac{v^2}{2g} = L_b \frac{v^2}{2g}$$

a = radius of pipe in feet,
r = radius of bend in feet,
v = velocity of flow in feet per second.

Values of L_b for different values of $\frac{a}{r}$ are as follows:

$\frac{a}{r}$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
L_b	0.131	0.138	0.158	0.206	0.294	0.440	0.661	0.977	1.40	1.98

Flow Through Straight Cylindrical Pipes

Q = discharge in cubic feet per second,
v = velocity of discharge in feet per second,
l = length of pipe in feet,
d = diameter of pipe in feet,
L_e = coefficient of loss at entrance. In general, the pipe is normal to the inner surface of the reservoir and then $L_e = 0.505$. For other cases see Loss at Entrance.
f = coefficient of friction, obtained from the table on page 145.

(1) Required the head in feet necessary to keep up a given flow of Q cubic feet per second in a clean iron pipe of given length l and diameter d.

The required head is

$$h \text{ (in feet)} = \frac{v^2}{2g}\left(1 + L_e + 4f\frac{l}{d}\right)$$

in which $$v = \frac{4Q}{\pi d^2}$$

(2) Required the velocity in the pipe, having given the head h and the length l and the diameter d of the pipe; also required the discharge Q in cubic feet per second.

The velocity in feet per second is:

$$v = \sqrt{\frac{2gh}{1 + L_e + 4f\frac{l}{d}}}$$

and after solving for v,

$$Q = \tfrac{1}{4}\pi d^2 v$$

Since the value of f depends on the unknown v as well as the known d, we may first put $f = 0.006$ for a trial approximation and solve for v; then take the value of f corresponding to this velocity and substitute again in the given formula for v. One trial is generally sufficient for ordinary accuracy.

(3) Required the proper diameter d for the pipe to discharge a given quantity Q cubic feet per second, having given the length of pipe and the head h.

The proper diameter in feet is

$$d = \sqrt[5]{\frac{(1 + L_e)\,d + 4fl}{2gh}\left(\frac{4Q}{\pi}\right)^2}$$

and d being solved for,

$$v = \frac{4\,Q}{\pi d^2}$$

Since the radical contains d, we must first assume a trial value for d, and taking $f = 0.006$, substitute in the above formula for the diameter. Having obtained a value for d, we solve for the velocity v. With the approximate values of d and v thus obtained, we find the corresponding new value of f from the table of friction, and then substitute again in the formulæ. One or two trials generally give sufficient accuracy.

Flow Through Very Long Pipes

When a pipe is very long (1000 feet or more), the head, velocity, or discharge, etc., may be calculated from the formulæ:

$$h = 4f\frac{l}{d}\,\frac{v^2}{2\,g} \qquad \text{(Chézy's formula)}$$

$$v = \frac{4\,Q}{\pi d^2}$$

$$h = \frac{64}{\pi^2} f \frac{lQ^2}{d^5 2\,g}$$

Notation same as in preceding section.

FLOW THROUGH OPEN CHANNELS

Bazin's Formula

The velocity of flow in a channel in feet per second is

$$v = \frac{87\sqrt{rs}}{0.552 + \dfrac{m}{\sqrt{r}}}$$

r = mean hydraulic radius in feet, which is found by dividing the area of the fluid cross-section in square feet by the wetted perimeter in feet (that is, the perimeter of the channel section in contact with the water),

s = slope of stream (that is, the difference in elevation between two points of the water surface divided by the distance between the two points measured along the surface),

m = coefficient of roughness, the values of which are given in the following table.

Character of channel	Value of m
Very smooth cement surfaces or planed boards..	0.06
Concrete, well-laid brick, unplaned boards......	0.16
Ashlar, good rubble masonry, poor brickwork...	0.46
Earth beds in perfect condition..................	0.85
Earth beds in ordinary condition................	1.30
Earth beds in bad condition covered with débris	1.75

Kutter's Formula

The velocity of flow in a channel in feet per second equals

$$v = \frac{41.65 + \dfrac{0.00281}{s} + \dfrac{1.811}{n}}{1 + \left(41.65 + \dfrac{0.00281}{s}\right)\dfrac{n}{\sqrt{r}}}\sqrt{rs}$$

where r and s are as in Bazin's formula.

Values for n, the coefficient of roughness, are as follows:

Character of channel	Value of n
Planed timber, glazed or enameled surfaces....	0.009
Smooth clean cement..........................	0.010
Unplaned timber, new well-laid brickwork.....	0.012
Smooth stonework, ordinary brickwork, iron...	0.013
Rough ashlar and good rubble masonry........	0.017
Firm gravel.................................	0.020
Earth in ordinary condition..................	0.025
Earth with stones, weeds, etc................	0.030
Earth or gravel in bad condition.............	0.035

FLOW OVER WEIRS

Contraction is **complete** when no edge of the weir is flush with the sides or bottom of the channel.

Contraction is **incomplete** when one or more sides of the weir have an interior border flush with the sides or bottom of the channel.

Francis' Formula

The flow over a weir in cubic feet per second is

$$Q = \tfrac{2}{3}\left[0.622\, h\left(b - \tfrac{1}{10}\, nh\right)\sqrt{2\, gh}\right]$$

in which

h = head in feet of water on weir,
b = width of weir in feet,
n = 2 for complete contraction,
n = 1 for one end of weir flush with side of channel,
n = 0 for both ends of weir flush with sides of channel.

Bazin's Formula for Weirs

For overfall-weirs with end contractions suppressed, the flow in cubic feet per second is

$$Q = \frac{2}{3} n \left[1 + 0.55 \left(\frac{h}{p+h}\right)^2\right] bh\sqrt{2\, gh}$$

in which the coefficient n has the value

$$n = 0.6075 + \frac{0.0148}{h}$$

h = depth in feet of water on weir,
b = width of weir in feet,
p = height in feet of the sill of the weir above the bottom of the channel of approach.

STRESSES IN PIPES AND CYLINDERS

Pressure in Pipes

The tensile stress in pounds per square inch in a pipe due to internal fluid pressure is:

For **thin pipes**, $p' = \frac{rp}{t}$

For **thick pipes or cylinders**,

$$p' = \frac{p(r + t)}{t}$$

r = inside radius of pipe in inches,
t = thickness of pipe in inches,
p = excess of internal over external pressure in pounds per square inch.

If S is the required factor of safety, then:

For **thin pipes**, $t = S\frac{rp}{P}$

For **thick pipes or cylinders**,

$$t = S\frac{rp}{P - pS}$$

in which r and p are as above, and

P = ultimate tensile strength of material of pipe (see Table of Strength of Materials).

Collapsing of Tubes

The collapsing pressure for Bessemer steel lap-welded tubes, for lengths greater than six diameters, is

$$\left.\begin{aligned} p &= 1000\left(1 - \sqrt{1 - 1600\frac{t^2}{d^2}}\right) \quad \text{when } \frac{t}{d} < 0.023 \\ &\text{or} \\ p &= 86670\frac{t}{d} - 1386 \qquad\qquad \text{when } \frac{t}{d} > 0.023 \end{aligned}\right\}$$

(Stewart's equations)

in which

p = excess of external over internal pressure in pounds per square inch,
d = outside diameter of tube in inches,
t = thickness of tube wall in inches.

FLOW OF FLUIDS

Flow of Air Through Apertures

The weight of air in pounds discharged per second from a reservoir into the atmosphere is

$$\left.\begin{aligned} M &= 0.53\,F\frac{p_1}{\sqrt{T_1}} \quad \text{when} \quad p_1 > 2\,p_a \\ &\text{or} \\ M &= 1.06\,F\sqrt{\frac{p_a(p_1 - p_a)}{T_1}} \quad \text{when} \quad p_1 < 2\,p_a \end{aligned}\right\} \text{Fliegner's equations}$$

p_1 = reservoir pressure in pounds per square inch absolute,
p_a = atmospheric pressure (14.7 pounds per square inch),
F = cross-section of aperture in square inches,
T_1 = absolute temperature of reservoir (degrees Fahr. + 459.6).

Flow of Steam Through Apertures

$M = 0.0165\, Fp_1^{0.97}$ (Grashof's formula)

$$\left.\begin{aligned} M &= \frac{Fp_1}{70} \quad \text{when} \quad p_1 > \frac{5}{3}p_2 \\ M &= \frac{Fp_2}{42}\sqrt{\frac{3\,(p_1 - p_2)}{2\,p_2}} \quad \text{when} \quad p_1 < \frac{5}{3}p_2 \end{aligned}\right\} \text{Napier's equations}$$

Grashof's formula applies when the final pressure is less than 58 per cent of the reservoir pressure.

M = pounds of steam discharged per second,
p_1 = reservoir pressure in pounds per square inch,
p_2 = final pressure in pounds per square inch,
F = cross-section of aperture in square inches.

Flow of Gas in Pipes

$$Q = 1000\sqrt{\frac{d^5h}{sl}} \qquad \text{(Molesworth)}$$

Q = quantity of gas in cubic feet per hour,
d = diameter of pipe in inches,
l = length of pipe in yards,
h = pressure in inches of water,
s = specific gravity of gas relative to air.

Flow of Air in Pipes

$$v = 114.5\sqrt{\frac{hd}{L}} \qquad \text{(Hawksley)}$$

v = velocity in feet per second,
h = head in inches of water,
d = diameter of pipe in inches,

L = length of pipe in feet,

$$Q = \frac{\pi}{4} \frac{d^2}{144} v$$

Q = quantity in cubic feet per second.

Flow of Compressed Air in Pipes

$$Q = 217.5 \sqrt{\frac{pd^5}{rL}}$$

$$d = 0.1161 \sqrt[5]{\frac{LQ^2 r}{p}} = 0.1161 \sqrt[5]{\frac{LQ_1^2}{pr}}$$

Q = volume in cubic feet per minute of compressed air, at 62° F.,

Q_1 = volume before compression, at 62° F.,

r = pressure in atmospheres,

p = difference in pressures in pounds per sq. inch, causing the flow,

d = diameter of pipe in inches,

L = length of pipe in feet.

Flow of Steam in Pipes

$$W = 87 \sqrt{\frac{w(p_1 - p_2)d^5}{L\left(1 + \frac{3.6}{d}\right)}} \qquad \text{(Babcock)}$$

W = weight of steam flowing in pounds per minute,

w = density in pounds per cubic foot of the steam at the entrance to the pipe,

p_1 = pressure in pounds per square inch at the entrance,

p_2 = pressure at exit,

d = diameter in inches,

L = length of pipe in feet.

ELECTRICITY

OHMIC RESISTANCE

The resistance of a uniform electric conductor at 0° Centigrade is given by the formula:

$$R \text{ (in ohms)} = \rho \frac{L}{A}$$

L = length of conductor in inches,
A = cross-section in square inches,
ρ = resistivity of conductor at 0° C., values of which are given in the following table.

TABLE OF RESISTIVITIES

(Resistivity is the resistance in ohms between any two opposite faces of a 1 inch cube of the material)*

Metal	Resistivity at 0° C.
Aluminium (annealed) . .	1.14×10^{-6}
Aluminium (commercial)	1.05×10^{-6}
Aluminium bronze	4.96×10^{-6}
Bismuth (compressed). . .	51.2×10^{-6}
Brass.	2.82×10^{-6}
Copper (drawn).	0.637×10^{-6}
Copper (annealed).	0.625×10^{-6}
German silver	8.23×10^{-6}
Gold (annealed).	0.803×10^{-6}
Iron (wrought).	3.82×10^{-6}
Lead (compressed).	7.68×10^{-6}
Magnesium.	1.72×10^{-6}
Mercury.	37.1×10^{-6}
Nickel (annealed).	4.89×10^{-6}
Platinum (annealed). . . .	3.53×10^{-6}
Silver (annealed).	0.575×10^{-6}
Tin.	5.16×10^{-6}
Tungsten.	$2. \times 10^{-6}$
Zinc (pressed).	2.28×10^{-6}

* This definition applies to English units and to the numerical values given in the table. In general, resistivity is the resistance of a unit cube.

The **resistance** of a conductor at **any temperature** is

$$R_2 = R_1 \frac{(1 + \alpha t_2)}{(1 + \alpha t_1)}$$

in which

R_1 = known resistance at a temperature t_1 degrees Centigrade,

R_2 = required resistance at a temperature t_2 degrees Centigrade,

α = temperature coefficient of electrical resistance, the value of which is given for different metals in the following table.

TEMPERATURE COEFFICIENTS OF ELECTRICAL RESISTANCE

Metal	Temp. coefficient (approx.) for 1° C.
Aluminium (commercial)	0.00435
Copper (annealed)	0.00388
German silver	0.00036
Gold (annealed)	0.00365
Iron (wrought)	0.00463
Mercury	0.00072
Platinum	0.00247
Silver	0.00377
Tungsten	0.00570
Zinc (pressed)	0.00365

Note. — The temperature coefficient of a material is its increase in resistance for each degree Centigrade rise in temperature, and it is expressed as a decimal fraction of the resistance at 0° C.

DATA ON ANNEALED COPPER WIRE

Gauge No. (B. & S.)	Diameter in mils			Cross-section of bare wire		Resistance in ohms per 1000 feet		Pounds per 1000 feet
	Bare	Single cotton covered	Double cotton covered	Circular mils	Sq. inches	Cold (25° C. =77° F.)	Hot (65° C. =149° F.)	
0000	460	....	....	212,000	0.166	0.0500	0.0577	641
000	410	....	....	168,000	0.132	0.0630	0.0727	508
00	365	....	....	133,000	0.105	0.0795	0.0917	403
0	325	....	....	106,000	0.0829	0.100	0.116	319
1	289	....	....	83,700	0.0657	0.126	0.146	253
2	258	....	....	66,400	0.0521	0.159	0.184	201
3	229	....	....	52,600	0.0413	0.201	0.232	159
4	204	211	....	41,700	0.0328	0.253	0.292	126
5	182	189	....	33,100	0.0260	0.319	0.369	100
6	162	169	174	26,300	0.0206	0.403	0.465	79.5
7	144	151	156	20,800	0.0164	0.508	0.586	63.0
8	128	136	141	16,500	0.0130	0.641	0.739	50.0
9	114	121	126	13,100	0.0103	0.808	0.932	39.6
10	102	108	112	10,400	0.00815	1.02	1.18	31.4
11	91	97	101	8,230	0.00647	1.28	1.48	24.9
12	81	87	91	6,530	0.00513	1.62	1.87	19.8
13	72	78	82	5,180	0.00407	2.04	2.36	15.7
14	64	70	74	4,110	0.00323	2.58	2.97	12.4
15	57	63	67	3,260	0.00256	3.25	3.75	9.86
16	51	56	59	2,580	0.00203	4.09	4.73	7.82
17	45	50	53	2,050	0.00161	5.16	5.96	6.20

DATA ON ANNEALED COPPER WIRE (*Continued*)

Gauge No. (B. & S.)	Diameter in mils			Cross-section of bare wire		Resistance in ohms per 1000 ft.		Pounds per 1000 feet
	Bare	Single cotton covered	Double cotton covered	Circular mils	Sq. inches	Cold (25° C. = 77° F.)	Hot (65° C. = 149° F.)	
18	40	45	48	1620	0.00128	6.51	7.51	4.92
19	36	39	43	1290	0.00101	8.21	9.48	3.90
20	32	36	40	1020	0.000802	10.4	11.9	3.09
21	28.5	32.5	36.5	810	0.000636	13.1	15.1	2.45
22	25.3	29.0	33.0	642	0.000505	16.5	19.0	1.94
23	22.6	26.6	30.6	509	0.000400	20.8	24.0	1.54
24	20.1	24.1	28.1	404	0.000317	26.2	30.2	1.22
25	17.9	21.9	25.9	320	0.000252	33.0	38.1	0.970
26	15.9	19.9	23.9	254	0.000200	41.6	48.0	0.769
27	14.2	18.2	22.2	202	0.000158	52.5	60.6	0.610
28	12.6	16.6	20.6	160	0.000126	66.2	76.4	0.484
29	11.3	15.3	19.3	127	0.0000995	83.4	96.3	0.384
30	10.0	14.0	18.0	101	0.0000789	105	121	0.304
31	8.9	12.9	16.9	79.7	0.0000626	133	153	0.241
32	8.0	11.9	15.9	63.2	0.0000496	167	193	0.191
33	7.1	11.1	15.1	50.1	0.0000394	211	243	0.152
34	6.3	10.3	14.3	39.8	0.0000312	266	307	0.120
35	5.6	9.6	13.6	31.5	0.0000248	335	387	0.0954
36	5.0	8.5	12.0	25.0	0.0000196	423	488	0.0757
37	4.5	....	....	19.8	0.0000156	533	616	0.0600
38	4.0	....	....	15.7	0.0000123	673	776	0.0476
39	3.5	....	....	12.5	0.0000098	848	979	0.0377
40	3.1	....	....	9.9	0.0000078	1070	1230	0.0299

Ohm's Law

$$I = \frac{E}{R} \qquad R = \frac{E}{I}$$

or

$$E = IR$$

I = current in amperes,
E = electromotive force in volts,
R = resistance in ohms.

The proper size of wire in circular mils for any direct current circuit on a two-wire system consisting of copper conductors is given by the formula:

$$\text{c.m.} = \frac{10.8 \times 2\,d \times I}{E}$$

or if the resistance is required,

$$r = \frac{E}{2\,d \times I}$$

where

r = resistance per foot of wire in ohms,
E = volts drop in line,
I = total line current in amperes,
d = distance from source to load in feet,
c.m. = cross-section of conductor in circular mils.

Resistance of Circuits

The resultant of several resistances in **series** equals

$$R = r_1 + r_2 + r_3 + \cdots$$

where r_1, r_2, r_3, etc., are the separate resistances.

The resultant of several resistances in **parallel** or **multiple** is given by the relation:

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots$$

R is the total or combined resistance; and r_1, r_2, r_3, etc., are the separate resistances.

Power and Energy in Direct Current Circuits

The power in watts expended in a resistance is

$$P = EI = I^2R$$

E = electromotive force in volts,
I = current in amperes,
R = resistance in ohms.

The **energy** transformed into heat in a time t seconds is

$$\epsilon = EIt = I^2Rt$$

when the current, I, is constant; or, if the current is variable, energy equals

$$\epsilon = \int_{t_1}^{t_2} i^2R\,dt$$

where i is the instantaneous value of the current, expressed as a function of t.

The **power** in any two-wire direct current circuit is

$$P \text{ (in watts)} = EI$$

where E is the volts between the terminals of the circuit and I is the current in amperes.

MOTORS AND GENERATORS

The **frequency** in cycles per second is given by the relation:

$$f = \frac{\text{R.P.M.}}{60} \times \frac{P}{2}$$

R.P.M. = speed in revolutions per minute,
P = number of poles.

Equations of Direct Current Motor

The **armature current** of a motor, during **starting**, is

$$I_a = \frac{E - e}{R_a + R_x}$$

in which

E = impressed voltage,
e = counter-electromotive force,
R_a = armature resistance in ohms,
R_x = resistance of grid or rheostat in series with armature.

At **full speed**,

$$I_a = \frac{E - e}{R_a}$$

$$e = K\phi f$$

$$E = I_a R_a + e = I_a R_a + K\phi f$$

$$I_a = \frac{E - K\phi f}{R_a}$$

$$f = \frac{E - I_a R_a}{K\phi}$$

f = frequency in cycles per second,*
ϕ = total field flux in magnetic lines, cutting armature conductors,
K = constant for any given machine. Its value is $\frac{4\,t}{10^8}$, where t is the number of armature turns in series.

* Frequency, in the case of a direct current machine, refers to the frequency of alternation in the armature windings, not, of course, in the external circuit.

Equations of Direct Current Generator

$$E = e - I_a R_a$$

e = generated voltage,
E = terminal voltage,
I_a = armature current in amperes,
R_a = armature resistance in ohms.

$$I_a = \frac{E}{R}$$

R = resistance of load in ohms.

$$E = RI_a$$

$$e = E + I_a R_a = I_a (R + R_a)$$

Torque

The torque of a dynamo in foot-pounds equals

$$T = KI\phi$$

where

ϕ = total field flux in magnetic lines, cutting armature conductors,
I = armature current in amperes,
K = constant term for any given dynamo. Its value is $K = \frac{2.348}{10^9} tP$, t being the number of armature turns in series, and P the total number of poles.

The torque of a motor in terms of the horsepower is

$$T = \frac{33{,}000 \text{ H.P.}}{2\pi n}$$

or solving for horsepower,

$$\text{H.P.} = \frac{2\pi T n}{33{,}000} = \frac{2\pi R F n}{33{,}000}$$

n = number of revolutions per minute,
T = torque in foot-pounds,
R = radius of pulley in feet,
F = turning force in pounds.

Induced Voltage

$$e = -\frac{N}{10^8}\frac{d\phi}{dt} \text{ volts}$$

N = number of turns.

If the turns cut across a uniform field, at right angles to the lines of force, then $\frac{d\phi}{dt}$ equals the number of lines cut per second. Otherwise, $\frac{d\phi}{dt}$ is the first derivative of ϕ in respect to t, ϕ being expressed as a function of t.

The **effective voltage** induced in the windings of a **generator, motor,** or **transformer,** etc., is given by the relation:

$$E = \frac{\sqrt{2}\,\pi f n\phi}{10^8} = \frac{4.44\, f n\phi}{10^8} \text{ volts}$$

This formula is generally quite accurate, being derived on the assumption of uniform flux distribution.

f = frequency in cycles per second,
ϕ = total number of lines of magnetic force,
n = effective number of turns. If all the turns are grouped in one coil, then n equals the total number of turns. Otherwise, if the winding is distributed over k electrical degrees (as in the armature of a motor or generator), then the effective number of turns is $n = N\frac{\sin\left(\frac{k}{2}\right)}{\frac{k}{2}}$, N being the total number of turns.

The **average induced voltage** of a dynamo is

$$E = \frac{4fn\phi}{10^8} \text{ volts}$$

where n is the number of armature turns in series.

Inductance

Inductance, L, is the number of interlinkages of flux with turns, per unit current,

$$L \text{ (henrys)} = \frac{N\phi}{10^8 I}$$

in which

N = number of turns,

I = current in amperes,

ϕ = number of lines of magnetic force interlinking with the turns.

The **theoretical unit** of inductance is the centimeter.

The **practical unit** of inductance is the henry, which equals 10^9 centimeters.

The **counter-electromotive force** in an inductive circuit is

$$e = -L\frac{di}{dt}$$

provided the inductance, L, is constant.

The **total voltage** consumed by an inductive circuit

$$E = ir + L\frac{di}{dt}$$

the inductance, L, being constant.

r is the resistance of the circuit in ohms, and $\frac{di}{dt}$ is the first derivative of i with respect to t, the current i being expressed as a function of t.

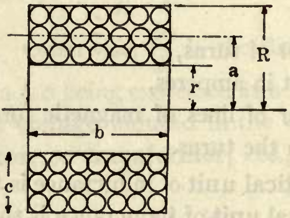
The **inductance** in henrys of an **air-core circular coil** is

$$L = \frac{0.366\left(\frac{l}{1000}\right)^2}{b + c + R} \times F' F''$$

$$F' = \frac{10\,b + 12\,c + 2\,R}{10\,b + 10\,c + 1.4\,R}$$

$$F'' = 0.5 \log_{10}\left(100 + \frac{14\,R}{2\,b + 3\,c}\right)$$

l = length of conductor in feet.



All other dimensions are in inches and as indicated in the diagram.

The **inductance,** L, of a **concentric cable** in henrys per 1000 feet is

$$L = \frac{3.048}{10^5} \times$$

$$\left\{\frac{1}{2} + 4.6 \log_{10} \frac{R}{r} + \frac{4.6\,R_0^4}{(R_0^2 - R^2)^2} \log_{10} \frac{R_0}{R} - \frac{1}{2}\,\frac{3\,R_0^2 - R^2}{(R_0^2 - R^2)}\right\}$$

where

r = radius of inner metallic conductor,

R = distance from center of cable to the inner surface of the outer metallic conductor,

R_0 = distance from center of cable to the outer surface of the outer metallic conductor.

The values of r, R, and R_0 must be expressed in the same units.

The **total inductance,** L, of a **two-wire transmission circuit** in henrys per 1000 feet is

$$L = \frac{3.048}{10^5}\left\{9.2\,\mu \log_{10}\frac{D-r}{r} + \mu_1\right\}$$

where

μ_1 = permeability of the metal conductor; for copper, $\mu_1 = 1$,

μ = permeability of medium separating wires; for air, $\mu = 1$,

D = distance between the two lines, measured from center to center,

r = radius of conductor, in same unit as D.

Capacity

The **unit of capacity** is the **farad.** Since the farad is very large, the **microfarad,** which is one-millionth of a farad, is used as the practical unit. The **theoretical unit** of capacity is the centimeter, 9×10^{11} centimeters being equal to 1 farad.

The **charge** of a condenser, Q, is measured in ampere-seconds or coulombs, and may be calculated by the formula:

$$Q = CE$$

from which

$$C = \frac{Q}{E}$$

and

$$E = \frac{Q}{C}$$

where

C = capacity in farads,

E = potential across the terminals of the condenser in volts.

The **capacity** of a **plate condenser** is

$$C = \frac{2248\,KA}{d \times 10^{10}} \text{ microfarads}$$

where

A = total area in square inches of **all** the dielectric sheets separating the condenser plates,

d = average thickness in inches of one sheet of the dielectric,

K = inductivity of the dielectric, average values of which are given in the following table for different materials.

Materials	Inductivity K
Air (at standard pressure)	1.00
Manilla paper	1.50
Paraffin, solid	2.00
Ebonite	2.50
India rubber	2.50
Shellac	3.00
Oil	3.00
Glass	3.10
Mica	6.00

Condensers in Parallel. When two or more condensers are connected in parallel, the resultant capacity, C, equals the sum of the separate capacities, thus

$$C = C_1 + C_2 + C_3 + \ldots$$

Condensers in Series. When two or more condensers of capacities C_1, C_2, C_3, etc., are connected in series, the resultant capacity is given by the formula:

$$C = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \ldots}$$

The **capacity,** C, of a **concentric cable** per 1000 feet in microfarads is

$$C = \frac{7.37}{1000 \log_{10} \frac{\rho_0}{\rho}}$$

in which

ρ = radius of inner metallic conductor,

ρ_0 = distance from center of cable to the inner surface of the outer metallic conductor, in the same unit as ρ.

The **capacity,** C, of a **two-wire transmission line** per 1000 feet in microfarads is given approximately by the formula:

$$C = \frac{3.68}{1000 \log_{10} \frac{D - r}{r}}$$

if the lines are not close to the ground.

D = distance between the two wires of the transmission line, measured from center to center,

r = radius of conductor, in same unit as D.

The **differential equations** of a **condenser** are

$$dq = i\,dt$$

$$q = \text{charge} = \int i\,dt$$

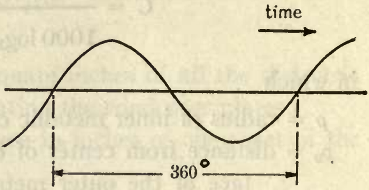
$$dq = c\,de$$

$$i = c\frac{de}{dt}$$

Alternating Current Circuits

The shape of the voltage or current wave produced by an alternator is, in general, nearly that of a **sine curve.** Alternating current calculations are, therefore, usually worked out on this assumption.

The number of cycles or complete waves per second is the **frequency** of the current, and the time required for the current to complete one cycle is a **period.**



The **average value** of the current or voltage is the average of all the ordinates of the curve of one half-wave. The **effective value** of an alternating current or voltage is the square root of the sum of the squares of the instantaneous values of a half-wave.

If E is the maximum voltage of a half-cycle of a sine wave,

$$\textbf{average voltage} = \frac{2}{\pi}E = 0.636\,E$$

$$\textbf{effective voltage} = \frac{1}{\sqrt{2}}E = 0.707\,E$$

Similarly, if the maximum current is I,

$$\textbf{average current} = \frac{2}{\pi}I = 0.636\,I$$

$$\textbf{effective current} = \frac{1}{\sqrt{2}}I = 0.707\,I$$

When the voltage reaches a definite value in the cycle sooner than the current reaches its corresponding value, the voltage and current are **out of phase** with each other; the voltage is said to be **leading,** and the current to be **lagging.** Phase difference is always expressed in degrees; a complete cycle equals 360 degrees.

Alternating Voltage and Current

$$I = \frac{E}{Z} \qquad Z = \frac{E}{I}$$

or

$$E = IZ$$

I = current in amperes,
E = electromotive force in volts,
Z = impedance in ohms.

Impedance and Reactance

r = resistance in ohms
x = reactance in ohms
z = impedance in ohms

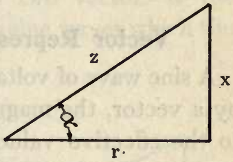
The relation between resistance, reactance, and impedance is the same as that between the three sides of a right triangle.

$$r = z \cos \alpha$$

$$x = z \sin \alpha$$

$$\alpha = \tan^{-1} \frac{x}{r}$$

$$z = \sqrt{r^2 + x^2}$$



Inductive Circuits

The **inductive reactance** in ohms is

$$x_L = 2 \pi f L$$

where f = frequency in cycles per second,
L = inductance in henrys.

The **impedance** in ohms is

$$z = \sqrt{r^2 + x_L^2} = \sqrt{r^2 + 4 \pi^2 f^2 L^2}$$

Circuits having Capacity

The **capacity reactance** in ohms is

$$x_C = -\frac{1}{2\pi f C}$$

where f = frequency in cycles per second,
C = capacity in farads.

The **impedance** in ohms is

$$z = \sqrt{r^2 + x_C^2} = \sqrt{r^2 + \frac{1}{4\pi^2 f^2 C^2}}$$

Circuits having Inductance and Capacity

The **reactance** in ohms is

$$x = x_L + x_C = 2\pi f L - \frac{1}{2\pi f C}$$

The **impedance** in ohms equals

$$z = \sqrt{r^2 + (x_L + x_C)^2}$$

Vector Representation of Sine Waves

A sine wave of voltage or current may be represented by a vector, the magnitude or length of which is equal to the effective value of the sine wave. It is sometimes more convenient to let the length of the vector equal the maximum value of the sine wave. The vector is generally denoted by a capital letter, with a dot directly beneath it; it is expressed in terms of its rectangular components, which determine the magnitude of the vector and its direction relative to the coördinate axes. Thus, the vector $\underset{\cdot}{E}$ is written

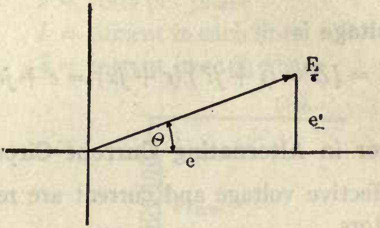
$$\underset{\cdot}{E} = e + je'$$

in which

$$j = \sqrt{-1}$$

where e denotes the horizontal or real component of the

vector, and e' the vertical or imaginary component. The imaginary unit, j, in the above equation, merely denotes the direction of measurement of e'.



The magnitude of $\dot{E}$ is

$$E = \sqrt{e^2 + e'^2}$$

and the angle θ which the vector $\dot{E}$ makes with the horizontal axis is

$$\theta = \tan^{-1} \frac{e'}{e}$$

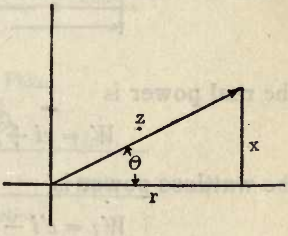
The angle in degrees between two vectors is the **phase difference** between the two sine waves which the vectors represent.

In vector notation, the **impedance** is

$$\dot{Z} = r + jx$$

and its magnitude is

$$Z = \sqrt{r^2 + x^2}$$



The **admittance** is

$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{r + jx} = \frac{r}{Z^2} - j\frac{x}{Z^2} = g + jb$$

where $g = \frac{r}{Z^2}$ = conductance,

$$b = -\frac{x}{Z^2} = \text{susceptance.}$$

The **current** equals

$$\underset{\cdot}{I} = \frac{\underset{\cdot}{E}}{\underset{\cdot}{Z}} = \underset{\cdot}{E}\underset{\cdot}{Y} = (e + je')\left(\frac{r}{Z^2} - j\frac{x}{Z^2}\right) = i + ji'$$

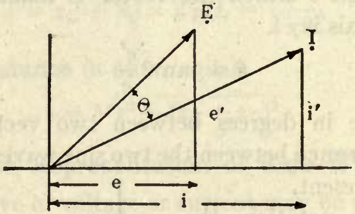
and the **voltage** is

$$\underset{\cdot}{E} = \underset{\cdot}{I}\underset{\cdot}{Z} = (i + ji')(r + jx) = e + je'$$

Power in Alternating Current Circuits

If the effective voltage and current are represented by the vectors

$$\underset{\cdot}{E} = e + je'$$
$$\underset{\cdot}{I} = i + ji'$$



the **real power** is

$$W = ei + e'i' = EI\cos\theta$$

the **wattless power** is

$$W_i = e'i - ei' = EI\sin\theta$$

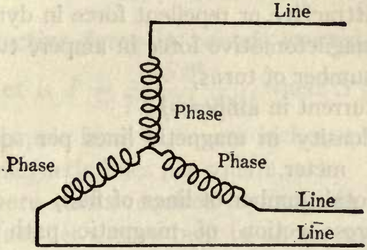
the **volt-amperes** equals EI.

The **power-factor** is the cosine of the angle between the voltage and current vectors,

$$\text{power-factor} = \cos\theta = \frac{ei + e'i'}{EI}$$

Balanced Three-phase Circuits

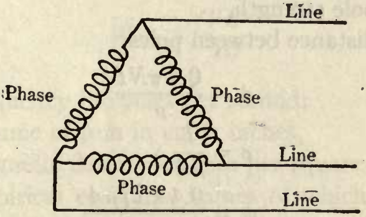
E = volts between lines
e = volts per phase
I = current in each line
i = current in each phase



Y connection

For Y-connections,

$$E = e\sqrt{3}; \quad e = \frac{E}{\sqrt{3}}; \quad \text{and} \quad I = i$$



Δ- connection

For Δ-connections,

$$E = e; \quad I = i\sqrt{3}; \quad \text{and} \quad i = \frac{I}{\sqrt{3}}$$

In either case, for non-inductive load, the power in watts is

$$W = \sqrt{3}\,EI$$

If the load is inductive, then the power is

$$W = \sqrt{3}\, EI \cos\theta$$

where $\cos\theta$ is the power-factor of the phase.

MAGNETISM

Equations of Magnetic Circuits

F = attractive or repellent force in dynes,
mmf = magnetomotive force in ampere turns,
N = number of turns,
I = current in amperes,
β = density in magnetic lines per square centimeter,
ϕ = total number of lines of flux,
A = cross-section of magnetic path in square centimeters,
μ = permeability,
H = intensity of field,
l = length of magnetic circuit in centimeters,
ρ = reluctance,
m = pole strength,
r = distance between poles.

$$\phi = \frac{0.4\,\pi NI}{\rho}$$

$$\rho = \frac{l}{\mu A}$$

$$\phi = \frac{0.4\,\pi NI\mu A}{l}$$

$$\beta = \frac{\phi}{A}$$

$$\beta = \frac{0.4\,\pi NI\mu}{l}$$

$$mmf = 0.4\,\pi NI$$

$$\mu = \frac{\beta}{H}$$

Magnets and Magnetic Fields

$$F = mH$$

$$F = \frac{mm'}{\mu r^2}$$

$$\phi = 4\pi m$$

The **attractive force** in pounds exerted by a **two pole magnet** is $P = \dfrac{SB^2}{72{,}134{,}000}$, where S is the total area of both pole faces in square inches, and B is the density in magnetic lines per square inch.

The **ampere-turns** required to maintain a flux density of B lines per square inch in an **air gap** is $IN = 0.313\,Bl$, in which l is the length of the gap in inches.

Hysteresis Loss

The power in watts lost in hysteresis is

$$W = k\frac{fVB^{1.6}}{10^7}$$

f = frequency in cycles per second,
V = volume of iron in cubic inches,
B = magnetic density in lines per square inch,
k = empirical constant, values of which are given in the following table.

Character of iron	Value of k
Silicon steel	0.0006 to 0.00075
Annealed sheet iron	0.0008 to 0.0011
Cast steel	0.010 to 0.012
Cast iron	0.013 to 0.017

Eddy Current Loss

The power in watts lost due to eddy currents in iron or steel laminations is approximately

$$W = \frac{0.00135}{10^7} f^2 l^2 B^2 V$$

f = frequency in cycles per second,
l = average thickness of lamination in inches,
B = magnetic density in lines per square inch,
V = volume of iron in cubic inches.

This formula holds for ordinary temperatures, and if the thickness of the lamination is not greater than 0.025 inch. In silicon steel, the eddy current loss is approximately $\frac{1}{3}$ of that given above.

STANDARD SATURATION CURVES

B = density in lines per square inch
AT/in. = ampere-turns per inch

Values of ampere-turns per inch for densities not included in the following tables may be determined approximately by interpolation. Thus, the AT/in. for silicon steel for B/sq.in. = 65,500 is

$$AT/\text{in.} = 4.5 + \frac{5500}{10{,}000}(6.4 - 4.5) = 5.5 \text{ (approx.)}$$

Silicon Steel		Annealed Sheet Iron	
Saturation curve		Saturation curve	
B	AT/in.	B	AT/in.
30,000	2.1	30,000	4
40,000	2.7	40,000	4.4
50,000	3.4	50,000	5
60,000	4.5	60,000	9
70,000	6.4	70,000	12
80,000	10	80,000	20
90,000	23	90,000	33
100,000	35	100,000	60
110,000	100	....	....
120,000	225	....	....
130,000	520	....	....
135,000	1000	....	....
140,000	2200	....	....
145,000	3770	....	....
150,000	5330	....	....
155,000	6900	....	....

Cast Steel		Cast Iron	
Saturation curve		Saturation curve	
B	AT/in.	B	AT/in.
50,000	11	5,000	8
60,000	15	10,000	12
70,000	20	15,000	17
80,000	29.5	20,000	23
90,000	50	25,000	30
100,000	105	30,000	43
105,000	165	35,000	60
....	....	40,000	85
....	....	45,000	110
....	....	50,000	145
....	....	55,000	190

AERONAUTICS

Balloons

For either rigid or non-rigid airships, in vertical equilibrium,

$$W + Vd' = Vd, \qquad W = V\,(d - d')$$

where W = gross weight in lb., exclusive of gas,
V = volume of gas bag, cu. ft.,
d, d' = densities of external air and internal gas, lbs. per cu. ft.

If P = absolute pressure, lb. per sq. ft.: T = absolute temperature (Fahr. temp. + 460), m = molecular weight of gas, then,

$$d' = P'm' \div 1544\,T', \qquad d = P \div 53.36\,T,$$

$$W = V\left(\frac{P}{53.36\,T} - \frac{P'm'}{1544\,T'}\right)$$

The lift (W) per cu. ft. of gas is $\frac{W}{V} = \frac{P}{53.36\,T} - \frac{P'm'}{1544\,T'}$, say $A - Bm'$, where P, T, P' and T' are standardized. The ratio of lifts per cu. ft. of two gases is then $r = (A - Bm_2') \div (A - Bm_1')$, under like conditions. The ratio of lifts per lb. of two gases under like conditions is $r\,\frac{m_1'}{m_2'}$.

Values of P and T are determined chiefly by the altitude (see page 202). Up to 10,000 ft., P decreases about 70 lb. from its sea-level value per 1000 ft. of

ascent. The value of T also decreases, and the seasonal variations in T decrease as altitude increases. The excess of P' over P is usually equivalent only to a few ounces per sq. in. This excess determines the tearing stress in the fabric (see page 153) which may be of the order of 100 lb. per lineal inch. Values of T' and T will differ somewhat, although the gas bag is painted with a non-absorbent coating.

Altitude control is most simply accomplished by dropping ballast (decreasing W) or by venting gas. The range of control is then greatest at the start and dampens down gradually by leakage or by the use of control. *Ballonets* (air bags), pumped full when it is desired to descend, prolong the control. There is a maximum allowable altitude (corresponding with a definite value of P) for every assigned set of conditions.

Resistance to flight in dirigible balloons at sea-level is given by $R = K\ V_0{}^n \sqrt{A^n}$, where R is in lb., A = total surface area of gas bag, sq. ft., V_0 = speed, ft. per sec., n is around 1.9 and K around 0.000015 for usual shapes from 4 to 8 diameters long. This must be somewhat increased to cover resistance to forward motion of car, structure, etc., The power required (thrust h.p. at the propeller) is $RV_0 \div 550$.

For minor items of resistance, if S = projected area on a transverse plane, sq. ft., and V is in miles per hr., $R' = K'\ S\ V^2$, where K' has the following values at sea-level:

Smooth wires normal to air,	$K' = .0026$
Cables normal to air,	$K' = .003$
Average for wheels uncovered,	$K' = .002$
covered,	$K' = .001$

Values of K, K', R and R' are directly proportional to the atmospheric density.

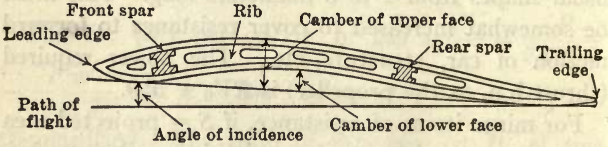
Airplanes

Wing Characteristics:

$$L = d\, k_L\, A V^2, \qquad D = d\, k_D\, A V^2,$$

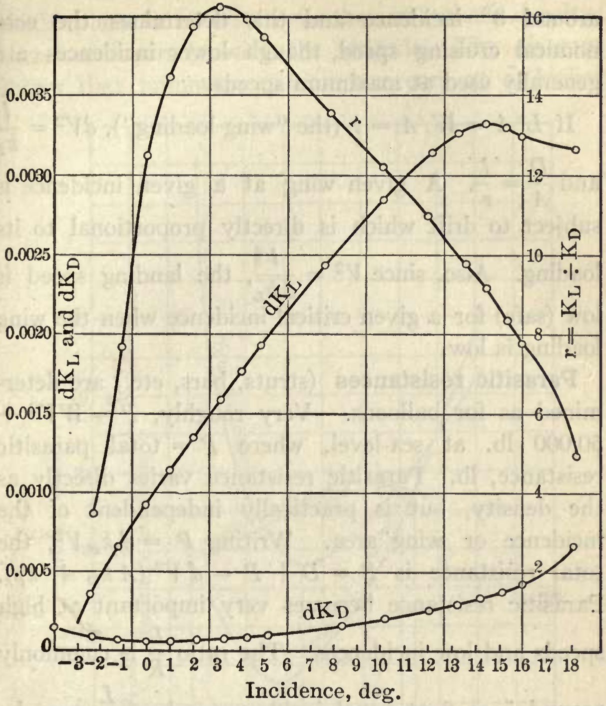
where d = atmospheric density,
L = lift, in lb., at right angles with the flight path,
D = drift or wing resistance, in lb., parallel with the flight path,
A = area of wing, sq. ft.,
k_L and k_D = lift and drift coefficients.

Values of k_L and k_D depend on size, shape, camber, etc. They are determined by wind tunnel experiments. The chief factor in determining the values of the coefficients is the **incidence** of the bottom chord of the wing against the flight path. Lift and drift are com-



ponents of an approximately normal force acting at the **center of pressure.** This force is mainly a suction on the upper face. The position of the center of pressure varies with the incidence.

As indicated in the diagram, maximum values of k_L are around 0.043 and maximum values of $r = k_L/k_D$



around 16. There is positive lift at slight negative incidences. The incidence of maximum lift (usually around 14°) is called the **critical incidence** or **burble point.** It should not be approached too closely. In horizontal flight, $L = W$, the weight of the plane, and speeds increase as incidences decrease. **Minimum** speed is determined by maximum k_L and should be low for safe landing. Least resistance (wing resistance alone) is realized at incidences around 0°, but here the lift is low. The best ratio of lift to drift occurs

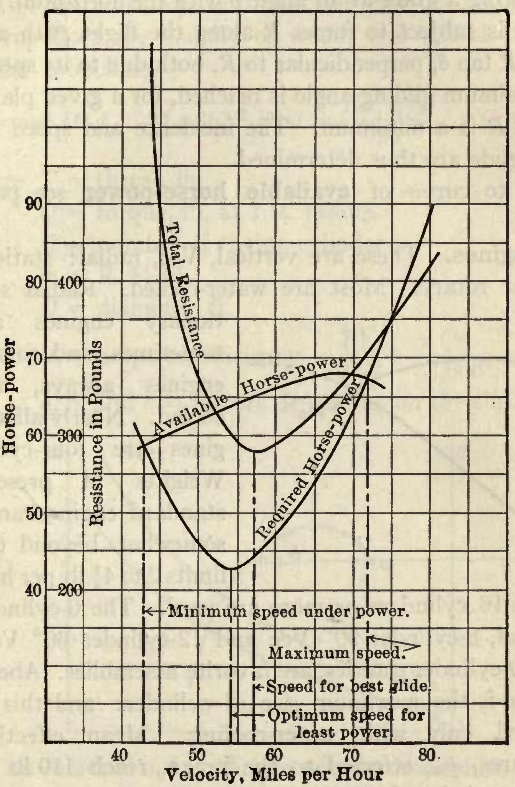
around 3° incidence and this determines the economical cruising speed, though lower incidences are generally used at maximum speeds.

If $L/A = W/A = l$ (the "wing loading"), $dV^2 = \frac{l}{k_L}$ and $\frac{D}{A} = \frac{l}{r}$. A given wing at a given incidence is subject to drift which is directly proportional to its loading. Also, since $V^2 = \frac{l}{d\,k_L}$, the landing speed is low (safe) for a given critical incidence when the wing loading is low.

Parasitic resistances (struts, bars, etc.) are determined as for balloons. Very roughly, $P = WV^2 \div 50{,}000$ lb. at sea-level, where P = total parasitic resistance, lb. Parasitic resistance varies directly as the density, but is practically independent of the incidence or wing area. Writing $P = d\,k_P\,V^2$, the total resistance is $R = D + P = d\,V^2\,(A\,k_D + k_P)$. Parasitic resistance becomes very important at high speeds and low incidences. The ratio $\frac{L}{R}$ is commonly around 5 or 6 at usual incidences, where $\frac{L}{D}$ may be as high as 12 to 16. For a given plane, greatest distance of flight is realizable when R is a minimum. Since h.p. $= H = R\,V \div 375$, efficiency, which is proportional to $\frac{L}{R}$, is also proportional to $\frac{W\,V}{H}$, which fraction really expresses $\frac{\text{effect}}{\text{cause}}$.

The **h.p. required** has a definite value, for a given plane, corresponding with each speed or incidence. At a given incidence, the power to propel any par-

ticular plane varies directly as the atmospheric density: therefore inversely with the altitude. The diagram shows that minimum resistance occurs at a higher



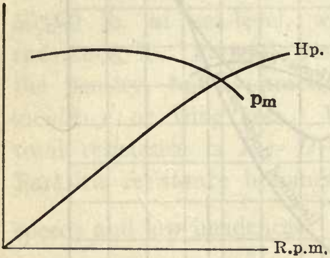
speed (lower incidence) than does minimum power required. The latter condition is the condition of maximum endurance (hours in the air). Since $W(= L)$ decreases during flight, the speeds of least power and

least resistance will vary: hence (whether the aim is greatest radius or greatest endurance) speed and incidence should vary during long horizontal flights.

During a **glide** at an angle θ with the horizontal, the plane is subject to forces R along the flight path and $L = R \tan \theta$, perpendicular to R, both due to its speed. A minimum gliding angle is reached, for a given plane, when R is a minimum. The incidence and speed for best glide are thus determined.

As to curve of **available horse-power** see page 188.

Engines. These are vertical, Vee, radial: stationary or rotary. Most are water-cooled. Radial stationary engines are sometimes, and rotary engines always, air-cooled. Nearly all engines are four-cycle. Weights of present standard engines range somewhat beyond the limits 2 to $4\frac{1}{2}$ lb. per h.p.



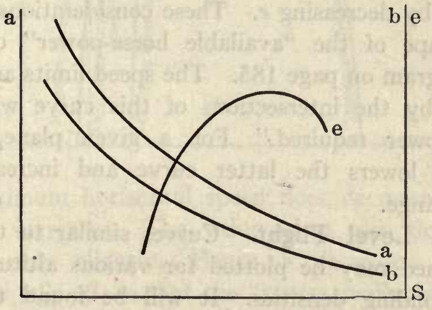
Up to 16 cylinders or more are used. The 6-cylinder vertical, 8-cylinder 90° Vee and 12-cylinder 60° Vee, with 9-cylinder rotaries, are favorite assemblies. About 30 h.p. is the maximum size of cylinder: and this is reached only with water-cooling. Mean effective pressures, p_m, referred to the brake, reach 110 lb. in water-cooled and 80 lb. in air-cooled types. Speeds may be as high as 2000 R. P. M., but a reduction gear between engine and propeller is commonly employed for speeds exceeding 1600 R. P. M. Mean effective pressures (at sea-level) are constant over a considerable

speed range. Horse-power varies directly with the speed, up to a rather high limit. For one single-acting four-cycle cylinder of d in. diameter, and s in. stroke, at n R. P. M., brake h.p. = $0.7854\ snd^2\ p_m \div (24 \times 33{,}000)$.

Propellors are usually of wood, two-bladed. For similar propellors,

$$T = an^2 D^4 d, \quad Q = bN^2 D^5 d = \frac{Ed^2\ s\ p_m}{96}, \quad e = c,$$

where T = thrust, lb.,
Q = torque, lb. at 1 ft. radius,
E = number of engine cylinders,
n = R. P. M.
D = diameter, ft.,
e = propeller efficiency = $\dfrac{TV}{375} \div \dfrac{\pi Qn}{33{,}000}$,
a, b, and c = factors depending on the slip.



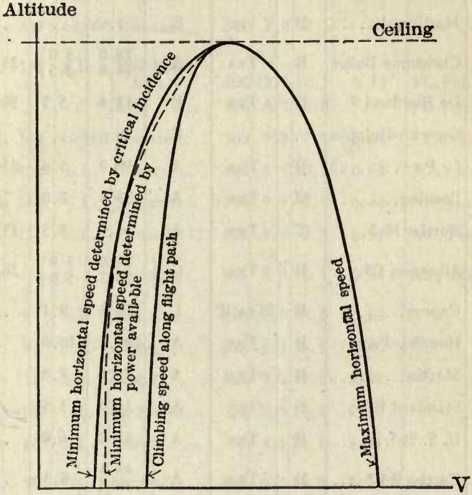
The values of a, b, and c are usually determined from wind tunnel experiments, and plotted on the base, $S = \dfrac{V}{nD}$, which may be called the **effective pitch ratio** and is equal in homologous units to $p\,(1 - s) \div D$, where p = pitch, s = slip.

In a given wheel, D is constant. Hence for constant efficiency, $\frac{V}{n}$ = constant. As will be shown, such constant ratio cannot be preserved: hence the propellor efficiency, which has a maximum value around 0.70, will seriously decrease at conditions other than those of the design.

Horizontal Power and Speed. Flying horizontally at sea-level, the engine and propellor torque remaining constant (because p_m is constant), $a_2n_2^2 = a_1n_1^2$ and n decreases as S decreases. Then $\frac{V_2}{V_1} = \frac{S_2}{S_1}\frac{n_2}{n_1}$, or decreasing V leads to decreasing S and n, and (if the propellor was designed for a maximum value of e at the highest value of V) it also leads to decreasing e. Reduced plane speed decreases engine output because it decreases n. It may further decrease propellor output by decreasing e. These considerations explain the shape of the "available horse-power" curve of the diagram on page 185. The speed limits are determined by the intersections of this curve with that for "power required." For a given plane, a low loading lowers the latter curve and increases the speed range.

High Level Flight. Curves similar to that last mentioned may be plotted for various altitudes and corresponding densities. It will be found that the power required for horizontal flight at a high altitude is less than that at sea-level at a high velocity and greater at a low velocity. Minimum power required is greater and is realized at a greater velocity, as the altitude increases. The power available from the engine decreases. The indicated power varies in

almost direct proportion with the atmospheric density. The engine friction losses remain about constant. Hence the torque power falls off rather more rapidly than the density, as the altitude increases. This alone might be sufficient to decrease the speed range, with increasing altitude. As a matter of observed fact,



the maximum horizontal speed does decrease as the altitude increases. The value of n also decreases, though only slightly. Hence S decreases and in general e falls off, so that the "power available" curve is decidedly lowered at high altitudes. Eventually there is reached an altitude (**ceiling** or **absolute ceiling**) at which only one speed is possible and above which flight is impossible.

Climbing. The best condition for climbing is that at which there is the greatest surplus of power avail-

CHARACTERISTICS

Service	Name	Form	Power	Nationality	Dimensions, ft. Spread	Chord	Length	Wing Area, sq. ft.
Scout.....	Spad 13C1......	B	TRA	F	26.3	..	20.4	215
"	Martinsyde.....	B	...	E	..	..	..	327
"	Christmas Bullet	B	TRA	A	28.0 14.0	5.0 2.5	21.0	170
Combat...	De Haviland 9..	B	TRA	E	42.4	5.5	30.8	434
Scout.....	Sopwith-Dolphin	..	..	E	..	..	..	263
Combat...	Le Pere........	B	TRA	A	39.0	5.6	25.4	392
" ...	Loening........	M	TRA	A	33.3	7.0	..	239
" ...	Curtiss 18-2....	T	TRA	A	31.9	3.5	23.3	309
Observation	Albatross C3....	B	TRA	G	38.8 36.7	5.9 5.6	26.0	407
Bomber...	Caproni........	B	2TRA,1P	I	76.8	9.1	..	1420
" ...	Handley-Page...	B	TRA	A	100.0	10.0	..	1648
" ...	Martin.........	B	TRA	A	71.4	7.8	..	1070
Training...	Standard E1....	B	TRA	A	24.0	3.5	..	153
" ...	U. S. Std.......	B	TRA	A	43.8	6.0	..	455
" ...	Curtiss R4 *	B	TRA	A	48.3 38.4	6.3	..	505
Mail plane.	Standard.......	B	TRA	A	31.4	6.0	26.6	337

B = biplane; TRA = tractor; F = French; A.I. = "Automotive Industries;" Age;" M = monoplane; Av. = "Aviation;" T = triplane; G = German; Æ =

able over power required. The amounts of *power required* for horizontal flight at this condition do not vary much with the altitude, for a given plane. Hence, low weight per h.p. of engine capacity favors rapid climb. The excesses of power available will be found to decrease in a straight-line relation with the altitude. But $H' = W c \div 33{,}000$, where H' = power available,

OF AIRPLANES

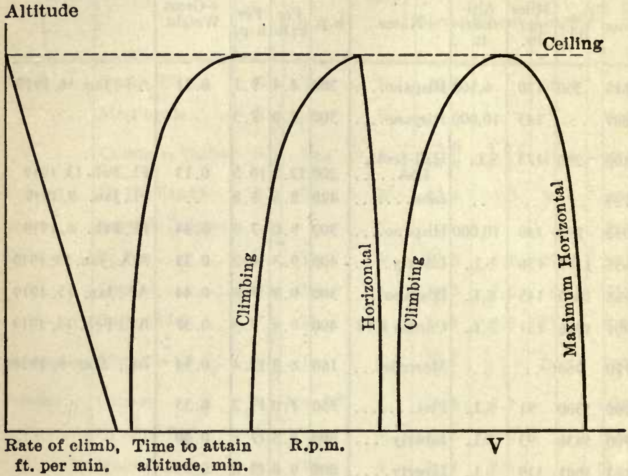
Weights, lb.		Speed		Engine		Loadings, lb.		Ratio, Useful Weight ÷Gross Weight	Reference
Gross	Useful	Miles per hr.	Altitude, ft.	Name	h.p.	Per sq.ft.	Per h.p.		
1815	556	130	6,500	Hispano....	200	8.4	9.1	0.31	A.I., Jan. 16, 1919
2289	..	143	10,000	Hispano....	300	7.0	7.5		
2100	280	175	S.L.	Hall-Scott, L6A.....	200	12.3	10.5	0.13	Fl., Feb. 13, 1919
3725	..	..	..	Lion.......	420	8.5	8.8	..	Fl., Jan. 9, 1919
2358	792	140	10,000	Hispano....	300	9.0	7.9	0.34	Fl., Feb. 6, 1919
3655	1187	136	S.L.	Liberty....	400	9.3	10.2	0.33	A.A., Jan. 13, 1919
2368	1040	145	S.L.	Hispano....	300	9.9	7.9	0.44	Av., Jan. 15, 1919
2901	1076	151	S.L.	Curtiss K12	400	9.4	7.3	0.37	Av., Feb. 15, 1919
2790	960	..	..	Mercedes...	160	6.8	12.4	0.34	Ae., Mar. 6, 1918
9900	3300	91	S.L.	Fiat.......	750	7.0	13.2	0.33	
13700	5430	93	S.L.	Liberty....	800	8.3	17.1	0.40	
9663	3801	119	S.L.	Liberty....	800	9.0	12.0	0.39	
1144	316	100	S.L.	Le Rhone...	80	7.5	14.3	0.30	
1950	600	68	S.L.	Curtiss.....	90	4.3	21.7	0.31	
3242	1017	90	S.L.	Curtiss.....	200	6.4	15.9	0.31	
2400	834	100	S.L.	Hispano I..	170	7.1	14.2	0.35	Av., Jan. 1, 1919

E = English; A = American; S.L. =sea-level; Fl. = "Flight;" A.A. = "Aerial Aeronautics; P = pusher; I = Italian.

* Also used as a 2 seater mailplane, when equipped with a 400 h.p. Liberty engine.

c = climb in ft. per min. Hence for a given plane, c bears a straight-line relation with the altitude. During the climb the plane speed (along its flight path) increases, the incidence being 6° to 8° at the start and usually 10° or 12° at the ceiling. The low speed at the start implies a low R. P. M. of the engine, which increases during the climb. The time-altitude

curve becomes horizontal at the ceiling. The **service ceiling** is that altitude at which the rate of climb becomes 100 ft. per min.



High ceiling is associated with rapid climbing. Both are favored by (*a*) low weight per h.p.; (*b*) low weight per sq. ft. of wing; (*c*) low parasitic resistance; (*d*) low engine friction.

Weights include structure, power plant, fuel and oil, crew and equipment. The two last mentioned rarely constitute over 35 per cent. of the total weight. The power plant weight includes radiator and water (for water-cooled engines), fuel and oil tanks, piping, and ignition apparatus. Water-cooled engines use about 0.55 lb. of fuel and 0.05 lb. of oil per h.p.-hr.: air-cooled, 50 per cent. more fuel and three times as much oil. On account of their lower weight per h.p., the latter engines have an advantage for short flights.

MEASUREMENT

English Weights and Measures

Length

1000 mils = 1 inch
12 inches = 1 foot
3 feet = 1 yard
5280 feet = 1 mile

4 inches = 1 hand
9 inches = 1 span
$2\frac{1}{2}$ feet = 1 pace
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards = 1 rod
1 knot or nautical mile = 6080.26 feet = $\frac{1}{3}$ league

7.92 inches = 1 link
25 links = 1 rod
100 links or 66 feet or 4 rods = 1 chain
10 chains = 1 furlong
8 furlongs = 1 mile

Surface

144 square inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
160 square rods = 1 acre
640 acres = 1 square mile

625 square links = 1 square rod
16 square rods = 1 square chain
10 square chains = 1 acre
640 acres = 1 square mile
36 square miles = 1 township

Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord
$24\frac{3}{4}$ cubic feet = 1 perch

Troy Weight

24 grains (gr.) = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

Avoirdupois Weight

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
25 pounds = 1 quarter (qr.)
4 quarters = 1 hundred weight (cwt.)
20 hundred weight (2000 pounds)
= 1 ton (T.)

Apothecaries' Weight

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (℔)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
36 bushels = 1 chaldron (ch.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bar.)
63 gallons = 1 hogshead (hhd.)

Apothecaries' Fluid Measure

60 minims = 1 fluid-drachm
8 fluid-drachms = 1 fluid-ounce
16 fluid-ounces = 1 pint
8 pints = 1 gallon

Circular Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
30 degrees = 1 sign (s)
12 signs, or 360 degrees = 1 circle (cir.)

English and Metric Conversion Tables

Length

1 millimeter	= 39.370 mils
	= 0.039370 inch
1 centimeter	= 0.39370 inch
	= 0.032808 foot
1 inch	= 2.5400 centimeters
	= 0.083333 foot
1 foot	= 30.480 centimeters
	= 0.30480 meter
1 yard	= 91.440 centimeters
	= 0.91440 meter
1 meter	= 39.370 inches
	= 3.2808 feet
	= 1.0936 yards
1 kilometer	= 3280.8 feet
	= 1093.6 yards
	= 0.62137 mile
1 mile	= 5280 feet
	= 1609.3 meters
	= 1.6093 kilometers

Surface

1 circular mil	= 0.78540 square mil
	= 0.00050671 square millimeter
1 square mil	= 1.2732 circular mils
	= 0.00064516 square millimeter
	= 0.000001 square inch
1 sq. millimeter	= 1973.5 circular mils
	= 1550.0 square mils
	= 0.0015500 square inch
1 sq. centimeter	= 197,350 circular mils
	= 0.15500 square inch
1 sq. inch	= 1,273,240 circular mils
	= 6.4516 square centimeters
1 sq. foot	= 929.03 square centimeters
	= 144 square inches

1 sq. yard	=	1296 square inches
	=	9 square feet
	=	0.0083613 are
	=	0.00020661 acre
1 sq. meter	=	1550.0 square inches
	=	10.764 square feet
	=	1.1960 square yards
1 are	=	1076.4 square feet
	=	100 square meters
1 acre	=	43,560 square feet
	=	4840 square yards
	=	4046.8 square meters
	=	0.40468 hectare
	=	0.0015625 square mile
1 hectare	=	107,640 square feet
	=	100 ares
	=	2.4711 acres
1 sq. kilometer	=	10,764,000 square feet
	=	1,196,000 square yards
	=	247.11 acres
	=	0.38610 square mile
1 square mile	=	27,878,400 square feet
	=	3,097,600 square yards
	=	640 acres
	=	2.5900 square kilometers

Volume

1 cu. centimeter	=	0.061024 cubic inch
	=	0.0021134 pint (liquid)
	=	0.0018162 pint (dry)
1 cu. inch	=	16.387 cubic centimeters
	=	0.017317 quart (liquid)
	=	0.014881 quart (dry)
	=	0.016387 liter or cubic decimeter
	=	0.0043291 gallon
	=	0.00057870 cubic foot
1 quart (liquid)	=	2 pints (liquid)
	=	946.33 cubic centimeters
	=	57.749 cubic inches
	=	0.94633 liter or cubic decimeter

1 quart (dry)	= 2 pints (dry)
	= 1101.2 cubic centimeters
	= 67.199 cubic inches
	= 0.038889 cubic foot
1 liter	= 1000 cubic centimeters
	= 61.024 cubic inches
	= 1.0567 quarts (dry)
	= 0.26418 gallon
1 cubic foot	= 1728 cubic inches
	= 28.317 liters or cubic decimeters
	= 0.028317 cubic meter
1 cubic yard	= 27 cubic feet
	= 0.76456 cubic meter
1 gallon	= 3785.3 cubic centimeters
	= 230.99 cubic inches
	= 4 quarts (liquid)
	= 3.7853 liters
	= 0.13368 cubic foot
1 cubic meter	= 35.315 cubic feet
	= 10 liters
	= 1.3080 cubic yards
	= 1 stere

NOTE.—Pints, quarts, and gallons in this table refer to U. S. measures.

Weight

1 milligram	= 0.015432 grain
	= 0.001 gram
1 grain *	= 64.799 milligrams
	= 0.0022857 ounce (av.)
1 gram	= 15.432 grains
	= 0.035274 ounce (av.)
	= 0.0022046 pound (av.)
1 ounce (av.)	= 437.50 grains
	= 28.350 grams
	= 0.062500 pound (av.)

* The troy grain and the apothecaries' grain are of the same weight as the avoirdupois grain.

1 ounce (troy)*	= 31.103 grams
1 pound (av.)	= 6999.97 grains
	= 453.59 grams
	= 16 ounces
	= 0.45359 kilogram
1 kilogram	= 35.274 ounces (av.)
	= 2.2046 pounds (av.)
1 ton (short)	= 2000 pounds (av.)
	= 907.18 kilograms
	= 0.89286 ton (long)
	= 0.90718 ton (metric)
1 ton (metric)	= 2204.6 pounds
	= 1000 kilograms
	= 1.1023 ton (short)
	= 0.98425 ton (long)
1 ton (long)	= 2240 pounds
	= 1.1200 ton (short)
	= 1.0160 ton (metric)

Force

Equivalents of force given below are dependent on the value of g, the acceleration of gravity. The standard value of g adopted by the International Committee on Weights and Measures, is $g = 980.665$, corresponding to 45° latitude and sea-level.

1 dyne	= 0.01574 grain
	= 0.00102 gram
	= 0.00007233 poundal
	= 0.000002248 pound (av.)
1 gram	= 980.6 dynes
	= 0.07093 poundal
1 poundal	= 13,825 dynes
	= 0.03108 pound
	= 0.01410 kilogram
1 pound	= 444,800 dynes
	= 32.17 poundals

* The apothecaries' ounce is of the same weight as the troy ounce.

1 kilogram	=	980600 dynes
	=	70.93 poundals

Storage of Water

1 acre-foot	=	325,800 gallons
	=	43,560 cu. feet
	=	1613 cu. yards
	=	1233 cu. meters
1 gallon	=	0.000003069 acre-foot
1 cu. foot	=	0.00002298 acre-foot
1 cu. yard	=	0.00062 acre-foot

Temperature

1 degree Centigrade	=	$\frac{9}{5}$ (= 1.8) degree Fahrenheit
1 degree Fahrenheit	=	$\frac{5}{9}$ (= 0.556) degree Centigrade
temperature Fahr.	=	$t_f = \frac{9}{5} t_c + 32$
temperature Cent.	=	$t_c = \frac{5}{9} (t_f - 32)$

Heat, Electric, and Mechanical Equivalents

Energy

1 erg	=	1 dyne-cm.
	=	0.0000001 joule
	=	0.00000007376 foot-pound
1 gram-centimeter	=	980.6 ergs
	=	0.00009806 joule
	=	0.00007233 foot-pound
1 joule	=	10,000,000 ergs
	=	0.7376 foot-pound
	=	0.2389 gram-calorie
	=	0.102 kilogram-meter
	=	0.0009480 B.t.u.
	=	0.0002778 watt-hour
1 foot-pound	=	13,560,000 ergs
	=	1.356 joules
	=	0.3239 gram-calorie
	=	0.1383 kilogram-meter
	=	0.001285 B.t.u.
	=	0.0003766 watt-hour
	=	0.0000005051 horsepower-hour

1 kilogram-meter	= 9.806 joules
	= 7.233 foot-pounds
	= 0.009296 B.t.u.
	= 0.002724 watt-hour
1 B.t.u.	= 1055 joules
	= 778.1 foot-pounds
	= 252 gram-calories
	= 107.6 kilogram-meters
	= 0.2930 watt-hour
	= 0.0003930 horsepower-hour
1 watt-hour	= 3600 joules
	= 2655.4 foot-pounds
	= 860 gram-calories
	= 3.413 B.t.u.
	= 0.001341 horsepower-hour
1 kilogram-calorie	= 4186 joules
	= 3088 foot-pounds
	= 426.9 kilogram-meters
	= 1.163 watt-hours
1 horsepower-hour	= 2,684,000 joules
	= 1,980,000 foot-pounds
	= 745.6 watt-hours

Power

1 erg per second	= 1 dyne-centimeter per second
	= 0.0000001 watt
1 gram-centimeter per second	= 0.00009806 watt
1 foot-pound per minute	= 0.02260 watt
	= 0.00003072 horsepower (metric)
	= 0.00003030 horsepower
1 watt	= 44.26 foot-pounds per minute
	= 6.119 kilogram-meters per minute
1 horsepower	= 33,000 foot-pounds per minute
	= 745.6 watts
	= 550 foot-pounds per second
	= 1.01387 horsepower (metric)
1 horsepower (metric)	= 32,550 foot-pounds per minute
	= 735.5 watts
	= 75 kilogram-meters per second
	= 0.9863 horsepower

1 kilowatt	= 44,256.7 foot-pounds per minute
	= 1.3597 horsepower (metric)
	= 1.341 horsepower

Electric Units

1 abvolt	= 10^{-8} volt
1 abampere	= 10 amperes
1 abohm	= 10^{-9} ohm

Pressure Equivalents

1 atmosphere (standard)	= 29.9212 inches of mercury at 32° F.
	= 760 millimeters of mercury at 32° F.
	= 33.901 feet of water at 39.1° F.
	= 14.6969 pounds per sq. inch
	= 2116.35 pounds per sq. foot
1 inch of mercury at 32° F.	= 0.491187 pound per sq. inch
	= 70.7310 pounds per sq. foot
	= 1.13299 feet of water at 39.1° F.
1 foot of water at 39.1° F.	= 0.8826 inch of mercury at 32° F.
	= 62.425 pounds per sq. foot
	= 0.4335 pound per sq. inch
	= 0.0295 atmosphere
1 pound on the sq. foot	= 0.016018 foot of water at 39.1° F.
1 pound on the sq. inch	= 2.307 feet of water at 39.1° F.

PRESSURE AND VOLUME CORRECTION, ETC.

Reduction of Barometer Readings to 0° C.

$$\text{corrected height } H_0 = H\left\{1 - \frac{(\beta - \alpha)\,t}{(1 + \beta t)}\right\}$$

H = observed height of barometer,

t = observed temperature of barometer in degrees Centigrade,

β = 0.0001818, the coefficient of cubical expansion of mercury,

α = coefficient of linear expansion of the material of the scale (0.0000085 for glass, 0.0000184 for brass).

Reduction of Gaseous Volumes to 0° C., and 1 Atmosphere Pressure

$$\text{corrected volume } v_0 = \left\{\frac{v}{1 + 0.00367\, t}\right\} \frac{p}{760}$$

v = observed volume,
t = observed temperature in degrees Centigrade,
p = pressure in millimeters of mercury.

Determination of Altitudes by the Barometer

For heights not exceeding 2000 feet, relative altitude is given by the approximate formula:

$$X \text{ (in feet)} = 52{,}500 \left\{1 + \frac{2\,(T + T_1)}{1000}\right\} \frac{H - H_1}{H + H_1}$$

X = vertical distance between the two stations,
T = Centigrade temperature at lower station,
T_1 = Centigrade temperature at upper station,
H = height of barometer at lower station reduced to 0° C.,
H_1 = height of barometer at upper station reduced to 0° C.

For any altitude,

$$X = 60{,}346 \{1 + 0.00256 \cos (2\theta)\} \left\{1 + 2\frac{(T + T_1)}{1000}\right\} \log_{10} \frac{H}{H_1}$$

in which θ = latitude in degrees.

Velocity of Sound

The velocity of sound in gases is

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

P = pressure,
ρ = density,
γ = ratio of specific heat at constant pressure to that at constant volume. (See Table, page 214.)

VELOCITY OF SOUND IN AIR AND WATER

Substance	Temperature, Degrees C.	Velocity in meters per second	Velocity in feet per second
Air	0	331.7	1088
Air	20	344	1129
Air	100	386	1266
Water	13	1441	4728
Water	19	1461	4794
Water	31	1505	4938

Geodetic and Astronomical Data

Velocity of light = 186,330 miles per second
= 299,870 kilometers per second

Equatorial radius of the earth * = 3963.339 miles
= 6378.388 kilometers

Polar semi-diameter of the earth * = 3949.992 miles
= 6356.909 kilometers

Mean distance from the earth to the moon = 238,854 miles
= 384,393 kilometers

Mean distance from the earth to the sun = 92,900,000 miles
= 149,500,000 kilometers

* U. S. C. & G. Survey.

PHYSICAL AND CHEMICAL CONSTANTS

INTERNATIONAL ATOMIC WEIGHTS (1919)

Element	Symbol	Atomic weight	Element	Symbol	Atomic weight
Aluminum	Al	27.1	Molybdenum	Mo	96.0
Antimony	Sb	120.2	Neodymium	Nd	144.3
Argon	A	39.88	Neon	Ne	20.2
Arsenic	As	74.96	Nickel	Ni	58.68
Barium	Ba	137.37	Niton	Nt	222.4
Bismuth	Bi	208.0	Nitrogen	N	14.01
Boron	B	11.0	Osmium	Os	190.9
Bromine	Br	79.92	Oxygen	O	16.00
Cadmium	Cd	112.40	Palladium	Pd	106.7
Cæsium	Cs	132.81	Phosphorus	P	31.04
Calcium	Ca	40.07	Platinum	Pt	195.2
Carbon	C	12.005	Potassium	K	39.10
Cerium	Ce	140.25	Praseodymium	Pr	140.9
Chlorine	Cl	35.46	Radium	Ra	226.0
Chromium	Cr	52.0	Rhodium	Rh	102.9
Cobalt	Co	58.97	Rubidium	Rb	85.45
Columbium *	Cb	93.1	Ruthenium	Ru	101.7
Copper	Cu	63.57	Samarium	Sa	150.4
Dysprosium	Dy	162.5	Scandium	Sc	44.1
Erbium	Er	167.7	Selenium	Se	79.2
Europium	Eu	152.0	Silicon	Si	28.3
Fluorine	F	19.0	Silver	Ag	107.88
Gadolinium	Gd	157.3	Sodium	Na	23.00
Gallium	Ga	69.9	Strontium	Sr	87.63
Germanium	Ge	72.5	Sulphur	S	32.06
Glucinum †	Gl	9.1	Tantalum	Ta	181.5
Gold	Au	197.2	Tellurium	Te	127.5
Helium	He	4.00	Terbium	Tb	159.2
Holmium	Ho	163.5	Thallium	Tl	204.0
Hydrogen	H	1.008	Thorium	Th	232.4
Indium	In	114.8	Thulium	Tm	168.5
Iodine	I	126.92	Tin	Sn	118.7
Iridium	Ir	193.1	Titanium	Ti	48.1
Iron	Fe	55.84	Tungsten	W	184.0
Krypton	Kr	82.92	Uranium	U	238.2
Lanthanum	La	139.0	Vanadium	V	51.0
Lead	Pb	207.20	Xenon	Xe	130.2
Lithium	Li	6.94	Ytterbium	Yb	173.5
Lutecium	Lu	175.0	Yttrium	Yt	88.7
Magnesium	Mg	24.32	Zinc	Zn	65.37
Manganese	Mn	54.93	Zirconium	Zr	90.6
Mercury	Hg	200.6			

* Columbium or Niobium (Nb). † Glucinum or Beryllium (Be).

WEIGHTS AND DENSITIES

Element	Temperature, Degrees C.*	Density in grams per cu. centimeter †
Aluminium	20	2.70
Antimony, pure	20	6.618
Compressed	20	6.691
Argon, liquid	−183	1.3845
Arsenic, crys	14	5.73
Barium	.....	3.78
Bismuth	20	9.781
Boron, crystal	.....	2.535
Amorphous	.....	2.45
Bromine, liquid	.....	3.12
Cadmium	20	8.648
Cæsium	20	1.873
Calcium	.....	1.54
Carbon, diamond	.....	3.52
Graphite	.....	2.25
Cerium	.....	7.02
Chlorine, liquid	−33.6	1.507
Chromium	20	6.92
Cobalt	21	8.71
Columbium	15	8.4
Copper	20	8.89
Erbium	.....	4.77
Fluorine, liquid	−200	1.14
Gallium	23	5.93
Germanium	20	5.46
Glucinum	.....	1.85
Gold	.....	9.33
Helium, liquid	−269	0.15
Hydrogen, liquid	−252	0.070
Indium	.....	7.28
Iridium	17	22.42
Iodine	20	4.940
Iron, pure	.....	7.86
Wrought	.....	7.8 to 7.9
Krypton, liquid	−146	2.16
Lanthanum	.....	6.15
Lead	20	11.347
Liquid	325	10.645
Lithium	20	0.534
Magnesium	.....	1.741
Manganese	.....	7.42
Mercury	0	13.596

WEIGHT AND DENSITIES

Element	Temperature, Degrees C.*	Density in grams per cu. centimeter †
Mercury	20	13.546
Liquid	−38.8	13.690
Solid	−38.8	14.193
Molybdenum	.....	9.01
Neodymium	.....	6.96
Nickel	.....	8.9
Nitrogen, liquid	−195	0.810
Osmium	.....	22.5
Oxygen	−184	1.14
Palladium	.....	12.16
Phosphorus, red	.....	2.20
Yellow	.....	1.83
Platinum	20	21.37
Potassium	20	0.870
Praseodymium	.....	6.475
Rhodium	.....	12.44
Rubidium	20	1.532
Ruthenium	0	12.06
Samarium	.....	7.7 to 7.8
Selenium	.....	4.3 to 4.8
Silicon, crys	20	2.42
Amorphous	15	2.35
Silver	20	10.503
Sodium	20	0.9712
Strontium	.....	2.50 to 2.58
Sulphur	.....	2.0 to 2.1
Tantalum	.....	16.6
Tellurium, amorphous	20	6.02
Thallium	.....	11.86
Thorium	17	12.16
Tin	.....	7.29
Titanium	18	4.5
Tungsten	.....	18.6 to 19.1
Uranium	13	18.7
Vanadium	.....	5.69
Xenon, liquid	−109	3.52
Zinc	20	7.13
Zirconium	.....	6.44

* Where temperature is not given, the value of density is for ordinary atmospheric temperatures.

† To reduce density in grams per cubic centimeter to pounds per cubic inch, multiply by 0.0361. To reduce density in grams per cubic centimeter to pounds per cubic foot, multiply by 62.4.

WEIGHTS AND DENSITIES (*Continued*)

Miscellaneous substances	Density in grams per cubic centimeter	Pounds per cubic foot
Agate	2.5 to 2.7	156 to 168
Asbestos	2.0 to 2.8	125 to 175
Aspahlt	1.1 to 1.5	69 to 94
Cement, set	2.7 to 3.0	170 to 190
Chalk	1.9 to 2.8	118 to 175
Clay	1.8 to 2.6	122 to 162
Coal, anthracite	1.4 to 1.8	87 to 112
Soft	1.2 to 1.5	75 to 94
Coke	1.0 to 1.7	62 to 105
Dolomite	2.84	177
Ebonite	1.15	72
Feldspar	2.55 to 2.75	159 to 172
Flint	2.63	164
Fluorite	3.18	198
Glass, common	2.4 to 2.8	150 to 175
Granite	2.64 to 2.76	165 to 172
Graphite	2.30 to 2.72	144 to 170
Hornblende	3.0	187
Ice	0.917	57.2
Ivory	1.83 to 1.92	114 to 120
Lime, mortar	1.65 to 1.78	103 to 111
Slaked	1.3 to 1.4	81 to 87
Limestone	2.68 to 2.76	167 to 171
Magnetite	4.9 to 5.2	306 to 324
Malachite	3.7 to 4.1	231 to 256
Marble	2.6 to 2.84	160 to 177
Mica	2.6 to 3.2	165 to 200
Paraffin	0.87 to 0.91	54 to 57
Pyrite	4.95 to 5.1	309 to 318
Quartz	2.65	165
Quartzite	2.73	170
Sandstone	2.14 to 2.36	134 to 147
Slate	2.6 to 3.3	162 to 205

WEIGHTS AND DENSITIES (*Continued*)

Woods *	Density in grams per cubic centimeter	Weight in pounds per cubic foot
Ash	0.65 to 0.85	40 to 53
Beech	0.70 to 0.90	43 to 56
Cedar	0.49 to 0.57	30 to 35
Cork	0.22 to 0.26	14 to 16
Elm	0.54 to 0.60	34 to 37
Fir	0.48 to 0.70	30 to 44
Lignum-vitæ	1.17 to 1.33	73 to 83
Mahogany	0.85	53
Maple	0.62 to 0.75	39 to 47
Oak	0.60 to 0.90	37 to 56
Pine, yellow	0.37 to 0.60	23 to 37
Pine, white	0.35 to 0.50	22 to 31
Poplar	0.35 to 0.50	22 to 31
Spruce	0.48 to 0.70	30 to 44
Walnut	0.64 to 0.70	40 to 43

* Seasoned and of average dryness.

Values for gases given below are for 0° Cent. (32° Fahr.) and a pressure of one atmosphere.

Gases	Density relative to air	Weight in grams per liter	Weight in pounds per cubic foot
Acetylene, C_2H_2	0.920	1.1620	0.07254
Air	1.000	1.2928	0.08071
Ammonia, NH_3	0.597	0.7706	0.04811
Carbon monoxide, CO	0.9672	1.2506	0.07807
Carbon dioxide, CO_2	1.5291	1.9768	0.12341
Ethane, C_2H_6	1.0494	1.3567	0.08470
Hydrochloric acid, HCl	1.2684	1.6398	0.10237
Hydrogen, H_2	0.0696	0.09004	0.005621
Hydrogen sulphide, H_2S	1.1895	1.5230	0.09508
Methane, CH_4	0.5576	0.7160	0.04470
Nitrous oxide, N_2O	1.5298	1.9777	0.12347
Nitric oxide, NO	1.0367	1.3402	0.08367
Nitrogen, N_2	0.9673	1.2514	0.07812
Oxygen, O_2	1.1053	1.4292	0.08922
Sulphur dioxide, SO_2	2.2639	2.9266	0.18271

WEIGHTS AND DENSITIES (*Continued*)

Liquids	Temperature, Degrees C.	Density in grams per cubic centimeter	Pounds per cubic foot
Acid, hydrochloric	....	1.20	74.8
Acid, nitric	....	1.22	76.0
Acid, sulphuric	....	1.84	116.5
Alcohol, ethyl	0	0.807	50.4
Alcohol, methyl	0	0.810	50.5
Carbolic acid	15	0.95 to 0.965	59.2 to 60 2
Carbon disulphide	0	1.293	80.6
Gasoline	....	0.66 to 0.69	41 to 43
Glycerine	0	1.26	78.6
Naphtha	15	0.665	41.5
Oil, linseed	15	0.942	58.8
Oil, olive	15	0.918	57.3
Petroleum	0	0.878	54.8
Turpentine	16	0.873	54.2
Water (freezing-point)	0	0.99987	62.417
(maximum density)	4	1.0000	62.425
(standard 62° F.)	16.7	0.99886	62.354
	20	0.99823	62.315
	100	0.9584	59.70
Water, sea (62° F.)	16.7	1.0260	63.976

MELTING AND BOILING POINTS OF ELEMENTS

Element	Melting point, Degrees C.	Melting point, Degrees F.	Boiling point at atmospheric pressure, Degrees C.	Boiling point at atmospheric pressure, Degrees F.
Aluminium	657	1215	1800	3272
Antimony	630	1166	1440	2624
Argon	−188	−306	−186	−303
Arsenic	(volatilizes)		(sublimes 450	842)
Barium	850	1562	.....	.....
Bismuth	269	516	1420	2590
Boron	2000 to 2500	3630 to 4530	(sublimes 3500	6330)
Bromine	−7.3	18.9	63	145.5
Cadmium	321	610	778	1432
Cæsium	26.4	79.5	670	1238
Calcium	780	1436	.....	.....
Carbon	4000	7230	.....	.....
Cerium	623	1153	.....	.....
Chlorine	−102	−151.6	−33.6	−28.5
Chromium	1520	2768	2200	3992
Cobalt	1480	2696	.....	.....
Columbium	1950	3542	.....	.....
Copper	1083	1982	2310	4190
Fluorine	−223	−369	−187	−305
Gallium	30.2	86.4	.....	.....
Glucinum	1430	2606	.....	.....
Gold	1063	1945	2530	4586
Helium	below −272	below −458	−268.8	−451
Hydrogen	−259	−434	−252.7	−423
Indium	155	311	1000	1830
Iodine	113	235	184.4	364
Iridium	2290	4150	2550	4610
Iron	1530	2786	2450	4442
Krypton	−169	−272	−151.7	−241.1
Lanthanum	810	1490	.....	.....
Lead	327	621	1525	2779
Lithium	186	367	1400	2552
Magnesium	633	1171	1120	2048
Manganese	1260	2320	1900	3452
Mercury	−38.87	−37.98	356.7	674
Molybdenum	2450	4440	3200	5790
Nickel	1452	2646	2330	4226

MELTING AND BOILING POINTS OF ELEMENTS
(Continued)

Element	Melting point		Boiling point at atmospheric pressure	
	Degrees C.	Degrees F.	Degrees C.	Degrees F.
Nitrogen	−210.5	−347	−195.7	−320
Osmium	2700	4890	.....	.....
Oxygen	−219	−362	−182.9	−297
Palladium	1549	2820	2540	4600
Phosphorus	44.1	111.4	287	549
Platinum	1755	3190	2450	4440
Potassium	62.5	144.5	758	1396
Praseodymium	940	1724	.....	.....
Radium	700	1290	.....	.....
Rhodium	1907	3465	2500	4530
Rubidium	38.5	111.3	696	1285
Ruthenium	1900	3450	2520	4570
Samarium	1350	2460	.....	.....
Selenium	217	423	690	1274
Silicon	1420	2588	3500	6330
Silver	961	1762	1955	3551
Sodium	97.0	206.6	750	1380
Strontium	900	1650	.....	.....
Sulphur	115	239	444.6	832.3
Tantalum	2910	5270	.....	.....
Tellurium	450	840	1390	3530
Thallium	301	574	1280	2790
Thorium	1690	3070	.....	.....
Tin	232	449.6	2270	4118
Titanium	1795	3440	.....	.....
Tungsten	3500	6330	3700	6690
Vanadium	1720	3130	.....	.....
Xenon	−140	−220	−109.1	−164.4
Zinc	418	784	918	1684
Zirconium	2300	4170	.....	.....

SPECIFIC HEATS

Element	Temperature, Degrees C.	Specific heat
Aluminium	16 to 100	0.2122
Antimony	17 to 92	0.0508
Arsenic, cryst	0 to 100	0.0861
Arsenic, amorphcus	0 to 100	0.0822
Barium	−185 to 20	0.068
Beryllium	0 to 100	0.425
Bismuth	20 to 100	0.0302
Bismuth, fluid	280 to 380	0.0363
Boron	0 to 100	0.307
Bromine, solid	−78 to −20	0.0843
Bromine, fluid	13 to 45	0.107
Cadmium	18 to 99	0.055
Cæsium	0 to 26	0.0482
Calcium	0 to 100	0.149
Carbon, graphite	11	0.160
Carbon, diamond	11	0.113
Cerium	0 to 100	0.0448
Chlorine, liquid	0 to 24	0.2262
Chromium	0	0.1039
Chromium	100	0.1121
Cobalt	15 to 100	0.1030
Copper	20 to 100	0.0936
Gallium, solid	12 to 23	0.079
Gallium, liquid	30 to 113	0.080
Germanium	0 to 100	0.0737
Gold	0 to 100	0.0316
Indium	0 to 100	0.0570
Iodine	9 to 98	0.0541
Iridium	18 to 100	0.0323
Iron, cast	20 to 100	0.1189
Iron, wrought	15 to 100	0.1152
Iron, wrought	0 to 1100	0.153
Iron, hard-drawn	20 to 100	0.1146
Lanthanum	0 to 100	0.0448
Lead	20 to 100	0.0305
Lead	300	0.0338
Lithium	0 to 100	1.093
Magnesium	20 to 100	0.2492
Manganese	20 to 100	0.1211
Mercury	20	0.0333
Molybdenum	20 to 100	0.0647
Nickel	18 to 100	0.109
Osmium	19 to 98	0.0311
Palladium	0 to 100	0.0592

SPECIFIC HEATS (*Continued*)

Element	Temperature, Degrees C.	Specific heat
Phosphorus, red	0 to 51	0.1829
Phosphorus, yellow	13 to 36	0.202
Platinum	0 to 100	0.0323
Potassium	−78 to 23	0.166
Rhodium	10 to 97	0.0580
Ruthenium	0 to 100	0.0611
Selenium, cryst	22 to 62	0.084
Selenium, amorphous	18 to 38	0.095
Silicon	57.1	0.1833
Silver	0 to 100	0.0559
Sodium	10	0.297
Sulphur, rhombic	0 to 54	0.1728
Sulphur, monoclinic	0 to 52	0.1809
Sulphur, liquid	119 to 147	0.235
Tantalum	58	0.036
Tellurium, cryst	15 to 100	0.0483
Thallium	20 to 100	0.0326
Thorium	0 to 100	0.0276
Tin	19 to 29	0.0552
Tin, molten	250	0.05799
Titanium	0 to 100	0.1125
Tungsten	0 to 100	0.0336
Uranium	0 to 98	0.028
Vanadium	0 to 100	0.1153
Zinc	0 to 100	0.0935
Zinc	300	0.1040
Zirconium	0 to 100	0.0660

Liquids	Temperature Degrees C.	Specific heat
Alcohol, ethyl	40	0.648
Alcohol, methyl	15 to 20	0.601
Benzene	10	0.340
Benzene	40	0.423
Brine (density 1.2)	−20	0.69
Glycerine	15 to 50	0.576
Oil, olive	7	0.47
Petroleum	21 to 58	0.511
Sea-water (density 1.024)	17.5	0.938
Turpentine	18	0.42
Water	0	1.0094
Water	20	1.0000
Water	100	1.0074

SPECIFIC HEATS (*Continued*)

Gases	Specific heat at constant pressure		Ratio, $\frac{c_p}{c_v}$, of the specific heat at constant pressure to that of constant volume	
	Temperature range, Degrees C.	Specific heat	Temperature range, Degrees C.	Ratio, $\frac{c_p}{c_v}$
Air	0 to 200	0.2375	0	1.402
Air	....	......	500	1.399
Alcohol, ethyl	108 to 220	0.4534	53	1.133
Alcohol	....	......	100	1.134
Ammonia	24 to 216	0.5125	0	1.317
Ammonia	....	......	100	1.277
Benzene	34 to 115	0.2990	60	1.403
Carbon monoxide	26 to 198	0.2426	0	1.403
Carbon monoxide	....	......	100	1.395
Carbon dioxide	11 to 214	0.2169	4 to 11	1.300
Carbon dioxide	....	......	500	1.260
Carbon disulphide	86 to 190	0.1596	3 to 67	1.205
Ethylene	....	0.4040	......	1.264
Hydrogen	12 to 198	3.4090	4 to 16	1.408
Methane	18 to 208	0.5929	11 to 30	1.316
Nitrogen	0 to 200	0.2438	......	1.410
Oxygen	13 to 207	0.2175	5 to 14	1.398
Oxygen	20 to 440	0.2240		

SPECIFIC HEATS (*Continued*)

Miscellaneous substances	Temperature, Degrees C.	Specific heat
Asbestos	20 to 98	0.195
Brass	14 to 98	0.0862
Charcoal	0 to 224	0.238
Glass, crown	10 to 50	0.161
Glass, flint	10 to 50	0.117
Granite	12 to 100	0.192
Ice	−21 to −1	0.502
India rubber	15 to 100	0.27 to 0.48
Limestone	15 to 100	0.216
Marble	0 to 100	0.21
Masonry	....	0.20
Paraffin wax, solid	0 to 20	0.694
Paraffin wax, fluid	60 to 63	0.712
Porcelain	15 to 1000	0.255
Quartz	20 to 98	0.191
Sandstone	....	0.22
Vulcanite	20 to 100	0.331

NOTE.—The specific heat of a material is the number of British Thermal Units necessary to raise the temperature of 1 pound of the material 1° Fahrenheit.

Coefficients of Linear Expansion of Solids

The length of a solid at any temperature is $l_t = l_o(1 + \alpha t)$, l_o being the known length at some given temperature, t the variation of temperature in degrees, and α the coefficient of linear expansion of the material. This formula holds approximately when the temperature interval is not large. The coefficient of **surface expansion** equals $2\,\alpha$; the coefficient of **cubical expansion** equals $3\,\alpha$.

COEFFICIENTS OF LINEAR EXPANSION (α)

The values given for α are the mean coefficients of expansion between 0° and 100° C., when some other temperature is not specified.

Elements	Temperature	Coefficient of linear expansion	
		For 1° C.	For 1° F.
Aluminium	....	0.00002220	0.00001233
Antimony	....	0.00001056	0.00000587
Arsenic	40	0.00000559	0.00000311
Bismuth	....	0.00001316	0.00000731
Cadmium	....	0.00003159	0.00001755
Carbon, diamond	40	0.00000118	0.00000066
Carbon, anthracite	40	0.00002078	0.00001154
Carbon, graphite	40	0.00000786	0.00000437
Cobalt	40	0.00001236	0.00000687
Copper	....	0.00001666	0.00000926
Gold	....	0.00001470	0.00000817
Indium	40	0.00004170	0.00002317
Iron, cast	40	0.00001061	0.00000589
Iron, annealed	....	0.00001089	0.00000605
Lead	....	0.00002709	0.00001505
Magnesium	40	0.00002694	0.00001497
Nickel	40	0.00001279	0.00000710
Osmium	40	0.00000657	0.00000365
Palladium	40	0.00001176	0.00000653
Phosphorus	0 to 40	0.00012530	0.00006961
Platinum	40	0.00000899	0.00000499
Potassium	0 to 50	0.00008300	0.00004611
Rhodium	40	0.00000850	0.00000472
Ruthenium	40	0.00000963	0.00000535
Selenium	40	0.00003680	0.00002044
Silicon	40	0.00000763	0.00000424
Silver	40	0.00001921	0.00001067
Sulphur	....	0.00011800	0.00006556
Tellurium	....	0.00003687	0.00002048
Thallium	40	0.00003021	0.00001678
Tin	....	0.00002296	0.00001276
Zinc	....	0.00002976	0.00001653

COEFFICIENTS OF LINEAR EXPANSION (α)
(*Continued*)

Miscellaneous substances	Temperature	Coefficient of linear expansion	
		For 1° C.	For 1° F.
Brass, cast	....	0.00001875	0.00001042
Brass, wire	....	0.00001930	0.00001072
Bronze	16.6 to 100	0.00001844	0.00001024
Ebonite	25 to 35	0.0000842	0.0000468
German silver	....	0.00001836	0.00001020
Glass, crown	....	0.00000897	0.00000498
Glass, flint	50 to 60	0.00000788	0.00000530
Glass, plate	....	0.00000891	0.00000495
Glass, tube	....	0.00000833	0.00000463
Gutta percha	20	0.0001983	0.0001102
Ice	−20 to −1	0.000051	0.000028
Marble	15 to 100	0.0000117	0.0000065
Paraffin wax	0 to 16	0.00010662	0.00005923
Paraffin wax	16 to 38	0.00013030	0.00007239
Porcelain	20 to 790	0.00000413	0.00000229
Quartz:			
Parallel to axis	0 to 80	0.00000797	0.00000443
Perpend. to axis	0 to 80	0.00001337	0.00000743

Woods *	Coefficient of linear expansion	
	For 1° C.	For 1° F.
(1) Along grain:		
Beech	0.00000257	0.00000143
Chestnut	0.00000649	0.00000361
Elm	0.00000565	0.00000314
Mahogany	0.00000361	0.00000201
Maple	0.00000638	0.00000347
Oak	0.00000492	0.00000273
Pine	0.00000541	0.00000301
Walnut	0.00000658	0.00000366
(2) Across grain:		
Beech	0.0000614	0.0000363
Chestnut	0.0000325	0.0000181
Elm	0.0000443	0.0000246
Mahogany	0.0000404	0.0000224
Maple	0.0000484	0.0000269
Oak	0.0000544	0.0000302
Pine	0.0000341	0.0000189
Walnut	0.0000484	0.0000269

* For temperature range 2° to 34° Cent.

PROPERTIES OF SATURATED STEAM

Tables condensed with permission from G. A. Goodenough's "Properties of Steam and Ammonia," published by Messrs. John Wiley and Sons.

Absolute pressure in inches of mercury	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
1	79.06	652	47.11	1095.0	1047.9	988.7	0.0915	1.9455	2.0370
2	101.17	338.9	69.16	1105.1	1036.0	974.3	0.1316	1.8474	1.9790
3	115.08	231.4	83.04	1111.4	1028.3	965.2	0.1561	1.7893	1.9454
4	125.44	176.5	93.37	1115.9	1022.5	958.3	0.1739	1.7478	1.9217
5	133.78	143.2	101.68	1119.6	1017.9	952.8	0.1380	1.7154	1.9034
6	140.80	120.7	108.69	1122.6	1013.9	948.1	0.1998	1.6888	1.8886
7	146.88	110.4	114.8	1125.2	1010.5	944.0	0.2098	1.6661	1.8760
8	152.26	92.1	120.2	1127.5	1007.4	940.4	0.2187	1.6464	1.8651
9	157.10	82.5	125.0	1129.6	1004.6	937.1	0.2265	1.6290	1.8556
10	161.50	74.8	129.4	1131.4	1002.1	934.1	0.2336	1.6134	1.8470
11	165.55	68.4	133.4	1133.1	999.7	931.3	0.2401	1.5992	1.8393
12	169.30	63.0	137.2	1134.7	997.5	928.8	0.2461	1.5862	1.8323
13	172.79	58.5	140.7	1136.1	995.5	926.4	0.2516	1.5742	1.8258
14	176.06	54.6	143.9	1137.5	993.6	924.1	0.2568	1.5630	1.8198
15	179.14	51.14	147.0	1138.8	991.7	922.0	0.2617	1.5526	1.8143
16	182.06	48.14	149.9	1140.0	990.0	920.0	0.2662	1.5429	1.8091
17	184.83	45.49	152.7	1141.1	988.3	918.1	0.2705	1.5337	1.8042
18	187.46	43.12	155.4	1142.1	986.7	916.2	0.2746	1.5250	1.7996
19	189.97	40.99	157.9	1143.1	985.2	914.4	0.2785	1.5168	1.7953
20	192.38	39.08	160.3	1144.1	983.8	912.7	0.2822	1.5089	1.7912

PROPERTIES OF SATURATED STEAM (*Continued*)

Absolute pressure in inches of mercury	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
21	194.68	37.34	162.6	1145.0	982.4	911.1	0.2858	1.5015	1.7873
22	196.89	35.75	164.8	1145.9	981.1	909.6	0.2892	1.4944	1.7835
23	199.03	34.29	167.0	1146.7	979.8	908.1	0.2924	1.4876	1.7800
24	201.09	32.95	169.0	1147.5	978.5	906.6	0.2955	1.4810	1.7766
25	203.08	31.71	170.1	1148.3	977.3	905.2	0.2986	1.4747	1.7733
26	205.00	30.57	173.0	1149.1	976.1	903.8	0.3015	1.4687	1.7702
27	206.87	29.51	174.8	1149.8	974.9	902.5	0.3043	1.4629	1.7671
28	208.67	28.53	176.6	1150.5	973.8	901.2	0.3070	1.4572	1.7642
29	210.43	27.61	178.4	1151.2	972.7	900.0	0.3096	1.4518	1.7614
in pounds per sq. inch									
14.7*	212.0	26.81	180.0	1151.7	971.7	898.8	0.3120	1.4469	1.7589
15	213.0	26.30	181.0	1152.2	971.2	898.1	0.3135	1.4438	1.7573
16	216.3	24.76	184.3	1153.4	969.1	895.8	0.3184	1.4337	1.7521
17	219.4	23.40	187.5	1154.6	967.1	893.5	0.3230	1.4242	1.7473
18	222.4	22.18	190.5	1155.7	965.2	891.4	0.3274	1.4153	1.7427
19	225.2	21.09	193.3	1156.7	963.4	889.3	0.3316	1.4068	1.7384
20	228.0	20.10	196.0	1157.7	961.7	887.3	0.3356	1.3987	1.7343
22	233.1	18.38	201.2	1159.6	958.4	883.6	0.3430	1.3837	1.7267
24	237.8	16.95	206.0	1161.3	955.3	880.1	0.3499	1.3698	1.7197
26	242.2	15.73	210.4	1162.8	952.4	876.8	0.3563	1.3570	1.7133
28	246.4	14.67	214.6	1164.3	949.7	873.7	0.3622	1.3452	1.7074

PROPERTIES OF SATURATED STEAM (*Continued*)

Absolute pressure in pounds per sq. in.	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
30	250.3	13.76	218.6	1165.7	947.1	870.7	0.3679	1.3340	1.7019
32	254.0	12.95	222.4	1166.9	944.6	867.9	0.3731	1.3236	1.6967
34	257.6	12.24	225.9	1168.1	942.2	865.2	0.3781	1.3137	1.6918
36	260.9	11.60	229.4	1169.2	939.9	862.7	0.3829	1.3044	1.6873
38	264.2	11.03	232.6	1170.3	937.7	860.2	0.3874	1.2956	1.6830
40	267.2	10.51	235.8	1171.3	935.5	857.8	0.3917	1.2871	1.6788
42	270.2	10.04	238.8	1172.2	933.5	855.5	0.3958	1.2791	1.6749
44	273.0	9.61	241.7	1173.2	931.5	853.3	0.3998	1.2714	1.6712
46	275.8	9.22	244.5	1174.0	929.6	851.2	0.4036	1.2640	1.6676
48	278.4	8.86	247.2	1174.8	927.7	849.1	0.4072	1.2570	1.6642
50	281.0	8.53	249.8	1175.6	925.9	847.1	0.4108	1.2501	1.6609
52	283.5	8.22	252.3	1176.4	924.1	845.1	0.4142	1.2436	1.6577
54	285.9	7.93	254.7	1177.1	922.4	843.2	0.4174	1.2373	1.6547
56	288.2	7.67	257.1	1177.8	920.7	841.4	0.4206	1.2311	1.6517
58	290.5	7.42	259.5	1178.5	919.0	839.5	0.4237	1.2252	1.6489
60	292.7	7.18	261.7	1179.1	917.4	837.8	0.4267	1.2195	1.6462
62	294.9	6.97	263.9	1179.7	915.8	836.0	0.4296	1.2139	1.6435
64	296.9	6.76	266.1	1180.3	914.3	834.3	0.4324	1.2085	1.6409
66	299.0	6.57	268.2	1180.9	912.7	832.7	0.4352	1.2032	1.6384
68	301.0	6.39	270.2	1181.5	911.2	831.1	0.4379	1.1981	1.6360

PROPERTIES OF SATURATED STEAM (*Continued*)

Absolute pressure in pounds per sq. in.	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
70	302.9	6.22	272.2	1182.0	909.8	829.5	0.4405	1.1931	1.6336
72	304.8	6.05	274.2	1182.5	908.3	827.9	0.4431	1.1883	1.6313
74	306.7	5.90	276.1	1183.0	906.9	826.4	0.4456	1.1835	1.6291
76	308.5	5.75	278.0	1183.5	905.5	824.9	0.4480	1.1789	1.6269
78	310.3	5.61	279.8	1184.0	904.2	823.4	0.4504	1.1744	1.6248
80	312.0	5.48	281.6	1184.4	902.8	821.9	0.4527	1.1700	1.6227
82	313.7	5.35	283.4	1184.9	901.5	820.5	0.4550	1.1657	1.6207
84	315.4	5.23	285.1	1185.3	900.2	819.1	0.4572	1.1615	1.6187
86	317.1	5.12	286.8	1185.7	898.9	817.7	0.4594	1.1574	1.6168
88	318.7	5.01	288.5	1186.1	897.7	816.3	0.4615	1.1534	1.6149
90	320.3	4.905	290.1	1186.5	896.4	815.0	0.4636	1.1495	1.6131
92	321.8	4.805	291.7	1186.9	895.2	813.7	0.4657	1.1456	1.6113
94	323.3	4.709	293.3	1187.3	894.0	812.4	0.4677	1.1419	1.6096
96	324.8	4.617	294.8	1187.7	892.8	811.1	0.4697	1.1381	1.6079
98	326.3	4.528	296.4	1188.0	891.6	809.8	0.4717	1.1345	1.6062
100	327.8	4.442	297.9	1188.4	890.5	808.6	0.4736	1.1309	1.6045
102	329.2	4.359	299.4	1188.7	889.3	807.4	0.4755	1.1274	1.6028
104	330.7	4.279	300.9	1189.0	888.2	806.1	0.4773	1.1239	1.6012
106	332.0	4.202	302.3	1189.4	887.1	804.9	0.4791	1.1205	1.5996
108	333.4	4.128	303.7	1189.7	885.9	803.8	0.4809	1.1172	1.5981

PROPERTIES OF SATURATED STEAM (*Continued*)

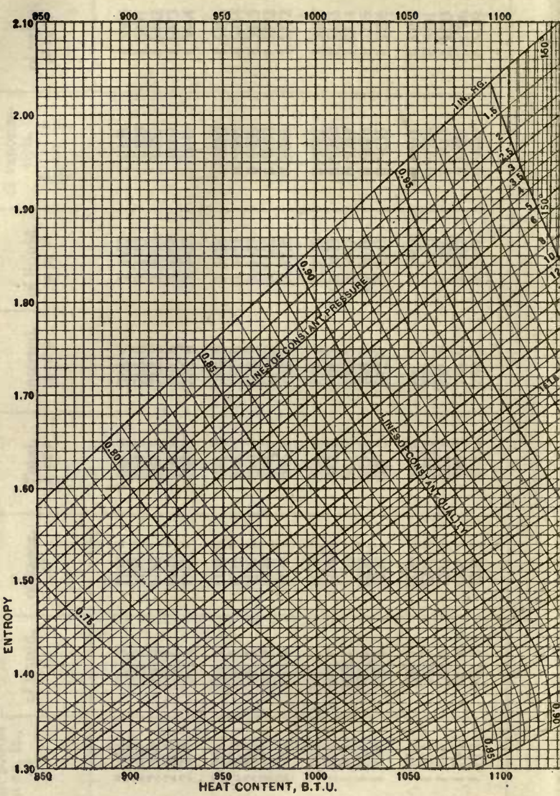
Absolute pressure in pounds per sq. in.	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
110	334.8	4.057	305.1	1190.0	884.8	802.6	0.4827	1.1138	1.5965
112	336.1	3.988	306.5	1190.3	883.7	801.4	0.4844	1.1106	1.5950
114	337.4	3.921	307.9	1190.6	882.7	800.3	0.4861	1.1074	1.5935
116	338.7	3.857	309.2	1190.8	881.6	799.2	0.4878	1.1043	1.5921
118	340.0	3.795	310.6	1191.1	880.6	798.0	0.4895	1.1012	1.5907
120	341.3	3.735	311.9	1191.4	879.5	796.9	0.4911	1.0982	1.5893
122	342.5	3.676	313.2	1191.6	878.5	795.8	0.4927	1.0952	1.5879
124	343.7	3.620	314.4	1191.9	877.5	794.8	0.4943	1.0922	1.5865
126	345.0	3.566	315.7	1192.1	876.4	793.7	0.4958	1.0894	1.5852
128	346.2	3.513	316.9	1192.4	875.4	792.6	0.4974	1.0865	1.5838
130	347 4	3.461	318.2	1192.6	874.4	791.6	0.4989	1.0836	1.5825
132	348.5	3.412	319.4	1192.9	873.5	790.5	0.5004	1.0808	1.5812
134	349.7	3.363	320.6	1193.1	872.5	789.5	0.5019	1.0781	1.5800
136	350.8	3.316	321.8	1193.3	871.5	788.5	0.5033	1.0754	1.5787
138	352.0	3.270	323.0	1193.5	870.5	787.4	0.5048	1.0727	1.5775
140	353.1	3.226	324.2	1193.7	869.6	786.4	0.5062	1.0700	1.5762
142	354.2	3.182	325.3	1193.9	868.6	785.4	0.5076	1.0674	1.5750
144	355.3	3.140	326.5	1194.1	867.7	784.5	0.5090	1.0648	1.5738
146	356.3	3.099	327.6	1194.3	866.8	783.5	0.5104	1.0623	1.5727
148	357.4	3.059	328.7	1194.5	865.8	782.5	0.5117	1.0598	1.5715

PROPERTIES OF SATURATED STEAM (*Continued*)

Absolute pressure in pounds per sq. in.	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
150	358.5	3.020	329.8	1194.7	864.9	781.6	0.5131	1.0573	1.5704
152	359.5	2.982	330.9	1194.9	864.0	780.6	0.5144	1.0548	1.5692
154	360.5	2.945	332.0	1195.1	863.1	779.7	0.5157	1.0524	1.5681
156	361.6	2.909	333.1	1195.3	862.3	778.7	0.5170	1.0500	1.5670
158	362.6	2.874	334.1	1195.5	861.4	777.8	0.5183	1.0476	1.5659
160	363.6	2.839	335.2	1195.7	860.5	776.9	0.5196	1.0453	1.5649
162	364.6	2.806	336.2	1195.8	859.6	776.0	0.5209	1.0429	1.5638
164	365.6	2.773	337.3	1196.0	858.7	775.1	0.5221	1.0406	1.5627
166	366.5	2.741	338.3	1196.2	857.9	774.2	0.5233	1.0384	1.5617
168	367.5	2.710	339.3	1196.3	857.0	773.3	0.5245	1.0361	1.5607
170	368.5	2.679	340.3	1196.5	856.2	772.4	0.5258	1.0339	1.5597
172	369.4	2.649	341.3	1196.6	855.3	771.5	0.5270	1.0317	1.5587
174	370.4	2.620	342.3	1196.8	854.5	770.6	0.5281	1.0295	1.5577
176	371.3	2.591	343.3	1196.9	853.6	769.8	0.5293	1.0274	1.5567
178	372.2	2.563	344.3	1197.1	852.8	768.9	0.5305	1.0252	1.5557
180	373.1	2.536	345.2	1197.2	852.0	768.0	0.5316	1.0231	1.5547
182	374.0	2.509	346.2	1197.4	851.2	767.2	0.5328	1.0210	1.5538
184	374.9	2.483	347.1	1197.5	850.4	766.4	0.5339	1.0189	1.5528
186	375.8	2.457	348.1	1197.6	849.5	765.5	0.5350	1.0169	1.5519
188	376.7	2.432	349.0	1197.8	848.7	764.7	0.5361	1.0148	1.5509

PROPERTIES OF SATURATED STEAM (*Continued*)

Absolute pressure in pounds per sq. in.	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of liquid, i'	of vapor, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s'
190	377.6	2.408	350.0	1197.9	847.9	763.9	0.5372	1.0128	1.5500
192	378.5	2.383	350.9	1198.0	847.1	763.0	0.5383	1.0108	1.5491
194	379.3	2.360	351.8	1198.1	846.3	762.2	0.5394	1.0089	1.5482
196	380.2	2.337	352.7	1198.2	845.6	761.4	0.5404	1.0069	1.5473
198	381.0	2.314	353.6	1198.4	844.8	760.6	0.5415	1.0049	1.5464
200	381.9	2.292	354.5	1198.5	844.0	759.8	0.5426	1.0030	1.5456
205	383.9	2.238	356.7	1198.7	842.1	757.8	0.5451	0.9983	1.5434
210	386.0	2.186	358.8	1199.0	840.2	755.9	0.5477	0.9936	1.5413
215	388.0	2.137	361.0	1199.2	838.3	754.0	0.5502	0.9890	1.5392
220	390.0	2.090	363.0	1199.5	836.5	752.1	0.5526	0.9846	1.5372
225	391.9	2.045	365.1	1199.7	834.6	750.2	0.5550	0.9802	1.5352
230	393.8	2.002	367.1	1199.9	832.8	748.3	0.5573	0.9760	1.5333
235	395.6	1.961	369.1	1200.1	831.0	746.5	0.5597	0.9717	1.5314
240	397.5	1.921	371.0	1200.3	829.3	744.7	0.5619	0.9676	1.5295
245	399.3	1.883	373.0	1200.5	827.5	742.9	0.5641	0.9635	1.5276
250	401.1	1.846	374.9	1200.6	825.8	741.2	0.5663	0.9595	1.5258
255	402.9	1.811	376.7	1200.8	824.1	739.5	0.5685	0.9556	1.5241
260	404.5	1.777	378.6	1201.0	822.4	737.7	0.5706	0.9517	1.5223
265	406.2	1.745	380.4	1201.1	820.7	736.0	0.5727	0.9479	1.5206
270	407.9	1.713	382.2	1201.2	819.1	734.4	0.5747	0.9442	1.5189



MOLLIER

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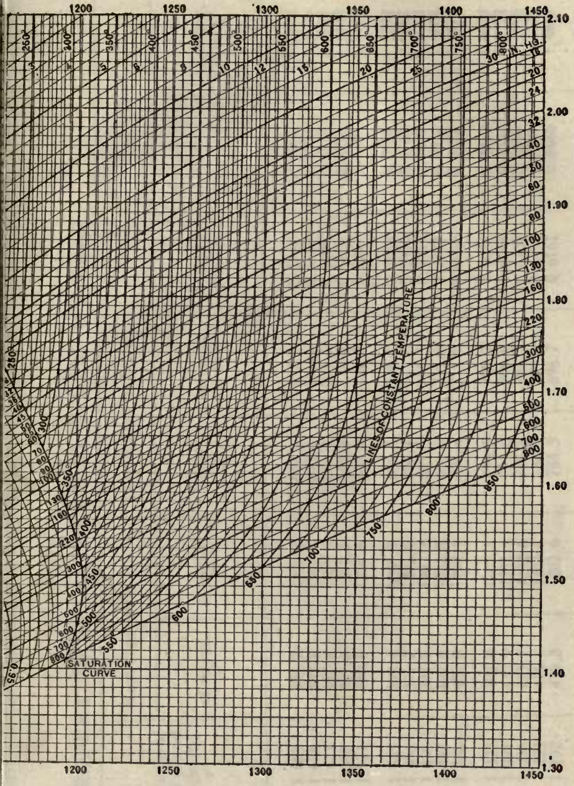

AM CHART

"Properties of Steam and Ammonia," published by
iley & Sons.

PROPERTIES OF SATURATED STEAM (*Continued*)

Absolute pressure in pounds per sq. in.	Temp. Fahr.	Volume of one pound in cu. ft., v''	Heat content in B.t.u.		Latent heat in B.t.u.		Entropy		
			of vapor, i'	of liquid, i''	Total, L or r	Internal, I or ρ	of liquid, n or s'	of vaporization, $\frac{L}{T}$ or $\frac{r}{T}$	of vapor, N or s''
275	409.6	1.683	383.9	1201.4	817.4	732.7	0.5767	0.9405	1.5172
280	411.2	1.654	385.7	1201.5	815.8	731.1	0.5787	0.9369	1.5156
285	412.8	1.625	387.4	1201.6	814.2	729.5	0.5806	0.9333	1.5139
290	414.4	1.598	389.1	1201.7	812.6	727.9	0.5826	0.9298	1.5123
295	415.9	1.571	390.8	1201.8	811.0	726.3	0.5845	0.9263	1.5108
300	417.5	1.545	392.4	1201.9	809.4	724.7	0.5863	0.9229	1.5092
310	420.5	1.496	395.7	1202.0	806.4	721.6	0.5900	0.9162	1.5062
320	423.4	1.450	398.9	1202.2	803.3	718.5	0.5935	0.9097	1.5032
330	426.3	1.407	402.0	1202.3	800.3	715.6	0.5970	0.9034	1.5004
340	429.1	1.366	405.0	1202.4	797.4	712.6	0.6004	0.8972	1.4976
350	431.9	1.327	408.0	1202.5	794.5	709.7	0.6036	0.8912	1.4949
360	434.6	1.291	410.9	1202.5	791.6	706.9	0.6068	0.8854	1.4922
370	437.2	1.256	413.7	1202.6	788.8	704.1	0.6100	0.8796	1.4896
380	439.8	1.223	416.5	1202.6	786.1	701.4	0.6130	0.8741	1.4871
390	442.3	1.192	419.3	1202.6	783.3	698.7	0.6161	0.8686	1.4847
400	444.8	1.162	422.0	1202.5	780.6	695.9	0.6190	0.8631	1.4821
450	456.5	1.033	434.8	1202.2	767.4	683.1	0.6329	0.8377	1.4706
500	467.2	0.928	446.6	1201.7	755.0	670.9	0.6455	0.8146	1.4601
600	486.5	0.770	468.0	1199.8	731.8	648.5	0.6679	0.7735	1.4414
700	503.4	0.656	487.1	1197.4	710.3	627.9	0.6874	0.7376	1.4250

TABLES

CIRCUMFERENCES AND AREAS OF CIRCLES

Diameter	Circumference	Area	Diameter	Circumference	Area
1	3.1416	0.7854	26	81.681	530.93
2	6 2832	3.1416	27	84.823	572.56
3	9.4248	7.0686	28	87.965	615.75
4	12.5664	12.5664	29	91.106	660.52
5	15.7080	19.635	30	94.248	706.86
6	18.850	28.274	31	97.389	754.77
7	21.991	38.485	32	100.53	804.25
8	25.133	50.266	33	103.67	855.30
9	28.274	63.617	34	106.81	907.92
10	31.416	78.540	35	109.96	962.11
11	34.558	95.033	36	113.10	1017.88
12	37.699	113.10	37	116.24	1075.21
13	40.841	132.73	38	119.38	1134.11
14	43.982	153.94	39	122.52	1194.59
15	47.124	176.71	40	125.66	1256.64
16	50.265	201.06	41	128.81	1320.25
17	53.407	226.98	42	131.95	1385.44
18	56.549	254.47	43	135.09	1452.20
19	59.690	283.53	44	138.23	1520.53
20	62.832	314.16	45	141.37	1590.43
21	65.973	346.36	46	144.51	1661.90
22	69.115	380.13	47	147.65	1734.94
23	72.257	415.48	48	150.80	1809.56
24	75.398	452.39	49	153.94	1885.74
25	78.540	490.87	50	157.08	1963.50

Note. — The surface of a sphere of given diameter may be found directly from the above table, since it is equal to the area of a circle of twice the diameter of the sphere.

CIRCUMFERENCES AND AREAS OF CIRCLES
(Continued)

Diameter	Circumference	Area	Diameter	Circumference	Area
51	160.22	2042.82	76	238.76	4536.46
52	163.36	2123.72	77	241.90	4656.63
53	166.50	2206.18	78	245.04	4778.36
54	169.65	2290.22	79	248.19	4901.67
55	172.79	2375.83	80	251.33	5026.55
56	175.93	2463.01	81	254.47	5153.00
57	179.07	2551.76	82	257.61	5281.02
58	182.21	2642.08	83	260.75	5410.61
59	185.35	2733.97	84	263.89	5541.77
60	188.50	2827.43	85	267.04	5674.50
61	191.64	2922.47	86	270.18	5808.80
62	194.78	3019.07	87	273.32	5944.68
63	197.92	3117.25	88	276.46	6082.12
64	201.06	3216.99	89	279.60	6221.14
65	204.20	3318.31	90	282.74	6361.73
66	207.34	3421.19	91	285.88	6503.88
67	210.49	3525.65	92	289.03	6647.61
68	213.63	3631.68	93	292.17	6792.91
69	216.77	3739.28	94	295.31	6939.78
70	219.91	3848.45	95	298.45	7088.22
71	223.05	3959.19	96	301.59	7238.23
72	226.19	4071.50	97	304.73	7389.81
73	229.34	4185.39	98	307.88	7542.96
74	232.48	4300.84	99	311.02	7697.69
75	235.62	4417.86	100	314.16	7853.98

POWERS, ROOTS, AND RECIPROCALS

Number	Square	Cube	Square root	Cube root	Reciprocal
1	1	1	1.000000	1.000000	1.0000000
2	4	8	1.414214	1.259921	.5000000
3	9	27	1.732051	1.442250	.3333333
4	16	64	2.000000	1.587401	.2500000
5	25	125	2.236068	1.709976	.2000000
6	36	216	2.449490	1.817121	.1666667
7	49	343	2.645751	1.912931	.1428571
8	64	512	2.828427	2.000000	.1250000
9	81	729	3.000000	2.080084	.1111111
10	100	1000	3.162278	2.154435	.1000000
11	121	1331	3.316625	2.223980	.0909091
12	144	1728	3.464102	2.289429	.0833333
13	169	2197	3.605551	2.351335	.0769231
14	196	2744	3.741657	2.410142	.0714286
15	225	3375	3.872983	2.466212	.0666667
16	256	4096	4.000000	2.519842	.0625000
17	289	4913	4.123106	2.571282	.0588235
18	324	5832	4.242641	2.620741	.0555556
19	361	6859	4.358899	2.668402	.0526316
20	400	8000	4.472136	2.714418	.0500000
21	441	9261	4.582576	2.758924	.0476190
22	484	10,648	4.690416	2.802039	.0454545
23	529	12,167	4.795832	2.843867	.0434783
24	576	13,824	4.898980	2.884499	.0416667
25	625	15,625	5.000000	2.924018	.0400000
26	676	17,576	5.099020	2.962496	.0384615
27	729	19,683	5.196152	3.000000	.0370370
28	784	21,952	5.291503	3.036589	.0357143
29	841	24,389	5.385165	3.072317	.0344828
30	900	27,000	5.477226	3.107233	.0333333
31	961	29,791	5.567764	3.141381	.0322581
32	1024	32,768	5.656854	3.174802	.0312500
33	1089	35,937	5.744563	3.207534	.0303030
34	1156	39,304	5.830952	3.239612	.0294118
35	1225	42,875	5.916080	3.271066	.0285714
36	1296	46,656	6.000000	3.301927	.0277778
37	1369	50,653	6.082763	3.332222	.0270270

POWERS, ROOTS, AND RECIPROCALS
(*Continued*)

Number	Square	Cube	Square root	Cube root	Reciprocal
38	1444	54,872	6.164414	3.361975	.0263158
39	1521	59,319	6.244998	3.391211	.0256410
40	1600	64,000	6.324555	3.419952	.0250000
41	1681	68,921	6.403124	3.448217	.0243902
42	1764	74,088	6.480741	3.476027	.0238095
43	1849	79,507	6.557439	3.503398	.0232558
44	1936	85,184	6.633250	3.530348	.0227273
45	2025	91,125	6.708204	3.556893	.0222222
46	2116	97,336	6.782330	3.583048	.0217391
47	2209	103,823	6.855655	3.608826	.0212766
48	2304	110,592	6.928203	3.634241	.0208333
49	2401	117,649	7.000000	3.659306	.0204082
50	2500	125,000	7.071068	3.684031	.0200000
51	2601	132,651	7.141428	3.708430	.0196078
52	2704	140,608	7.211103	3.732511	.0192308
53	2809	148,877	7.280110	3.756286	.0188679
54	2916	157,464	7.348469	3.779763	.0185185
55	3025	166,375	7.416199	3.802953	.0181818
56	3136	175,616	7.483315	3.825862	.0178571
57	3249	185,193	7.549834	3.848501	.0175439
58	3364	195,112	7.615773	3.870877	.0172414
59	3481	205,379	7.681146	3.892997	.0169492
60	3600	216,000	7.745967	3.914868	.0166667
61	3721	226,981	7.810250	3.936497	.0163934
62	3844	238,328	7.874008	3.957892	.0161290
63	3969	250,047	7.937254	3.979057	.0158730
64	4096	262,144	8.000000	4.000000	.0156250
65	4225	274,625	8.062258	4.020726	.0153846
66	4356	287,496	8.124038	4.041240	.0151515
67	4489	300,763	8.185353	4.061548	.0149254
68	4624	314,432	8.246211	4.081655	.0147059
69	4761	328,509	8.306624	4.101566	.0144928
70	4900	343,000	8.366600	4.121285	.0142857
71	5041	357,911	8.426150	4.140818	.0140845
72	5184	373,248	8.485281	4.160168	.0138889
73	5329	389,017	8.544004	4.179339	.0136986

POWERS, ROOTS, AND RECIPROCALS (*Continued*)

Number	Square	Cube	Square root	Cube root	Reciprocal
74	5476	405,224	8.602325	4.198336	.0135135
75	5625	421,875	8.660254	4.217163	.0133333
76	5776	438,976	8.717798	4.235824	.0131579
77	5929	456,533	8.774964	4.254321	.0129870
78	6084	474,552	8.831761	4.272659	.0128205
79	6241	493,039	8.888194	4.290840	.0126582
80	6400	512,000	8.944272	4.308870	.0125000
81	6561	531,441	9.000000	4.326749	.0123457
82	6724	551,368	9.055385	4.344482	.0121951
83	6889	571,787	9.110434	4.362071	.0120482
84	7056	592,704	9.165151	4.379519	.0119048
85	7225	614,125	9.219545	4.396830	.0117647
86	7396	636,056	9.273619	4.414005	.0116279
87	7569	658,503	9.327379	4.431048	.0114943
88	7744	681,472	9.380832	4.447960	.0113636
89	7921	704,969	9.433981	4.464745	.0112360
90	8100	729,000	9.486833	4.481405	.0111111
91	8281	753,571	9.539392	4.497941	.0109890
92	8464	778,688	9.591663	4.514357	0108696
93	8649	804,357	9.643651	4.530655	.0107527
94	8836	830,584	9.695360	4.546836	.0106383
95	9025	857,375	9.746794	4.562903	.0105263
96	9216	884,736	9.797959	4.578857	.0104167
97	9409	912,673	9.848858	4.594701	.0103093
98	9604	941,192	9.899495	4.610436	.0102041
99	9801	970,299	9.949874	4.626065	.0101010
100	10,000	1,000,000	10.000000	4.641589	.0100000

Logarithmic Cross-section Paper

Cross-section paper the rulings of which are proportional to the logarithms of the scale is called logarithmic cross-section paper. This paper is most convenient for plotting equations with constant exponents since they are straight lines on logarithmic paper while

they are curves if plotted on ordinary graph paper, in which case they must be plotted point by point.

The chief use of logarithmic cross-section paper is for plotting equations of the form:

$$y = ax^n$$

If two pairs of values of x and y are known, the corresponding points may be plotted on logarithmic paper and joined by a straight line. The value of the coefficient a is equal to the intercept of this line on the Y-axis, and the value of the exponent n is equal to the slope of the line (that is, the tangent of the angle which the line makes with the X-axis). The reason for this is that plotting on logarithmic paper is equivalent to taking logarithms, in which case we would obtain:

$$\log y = \log a + n \log x$$

which is the equation of a straight line, $\log a$ being the intercept and n the slope.

In case the values of a and n are known, that is, the intercept and the slope, we may plot the line, and from it obtain any pair of values of x and y.

Use of Logarithm Tables

Every logarithm consists of two parts: a positive or negative whole number called the **characteristic,** and a **positive** fraction, called the **mantissa.** The mantissa is always expressed as a decimal, and is the part which is given in the tables.

To find the common logarithm of a given number:

If the number is greater than 1, the characteristic of the logarithm is one unit less than the number of figures on the left of the decimal point.

If the number is less than 1, the characteristic of the logarithm is negative, and one unit more than the number of zeros between the decimal point and the first significant figure of the given number.

Thus,

$$\begin{aligned}
\log 20.6 &= 1.3139 \qquad \text{(base 10)} \\
\log 2.06 &= 0.3139 \\
\log 0.206 &= 0.3139 - 1 = 9.3139 - 10 \\
\log 0.0206 &= 0.3139 - 2 = 8.3139 - 10
\end{aligned}$$

To find the number corresponding to a given common logarithm:

If the characteristic of a given logarithm is positive, the number of figures in the integral part of the corresponding number is one more than the number of units in the characteristic.

If the characteristic is negative, the number of zeros between the decimal point and the first significant figure of the corresponding number is one less than the number of units in the characteristic.

COMMON LOGARITHMS OF NUMBERS
(Base 10)

N	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088

COMMON LOGARITHMS OF NUMBERS
Continued)

N	0	1	2	3	4	5	6	7	8	9
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	41 664	41 681	41 697	41 714	41 731	41 747	41 764	41 780	41 797	41 814
262	41 830	41 847	41 863	41 880	41 896	41 913	41 929	41 946	41 963	41 979
263	41 996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700

COMMON LOGARITHMS OF NUMBERS
(Continued)

N	0	1	2	3	4	5	6	7	8	9
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	45 163
283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
308	48 855	48 869	48 883	48 897	48 911	48 926	48 940	48 954	48 968	48 982
309	48 996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	50 651	50 664	50 678	50 691	50 705	50 718	50 732	50 745	50 759	50 772
322	50 786	50 799	50 813	50 826	50 840	50 853	50 866	50 880	50 893	50 907
323	50 920	50 934	50 947	50 961	50 974	50 987	51 001	51 014	51 028	51 041
324	51 055	51 068	51 081	51 095	51 108	51 121	51 135	51 148	51 162	51 175

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
327	51 455	51 468	51 481	51 495	51 508	51 521	51 534	51 548	51 561	51 574
328	51 587	51 601	51 614	51 627	51 640	51 654	51 667	51 680	51 693	51 706
329	51 720	51 733	51 746	51 759	51 772	51 786	51 799	51 812	51 825	51 838
330	51 851	51 865	51 878	51 891	51 904	51 917	51 930	51 943	51 957	51 970
331	51 983	51 996	52 009	52 022	52 035	52 048	52 061	52 075	52 088	52 101
332	52 114	52 127	52 140	52 153	52 166	52 179	52 192	52 205	52 218	52 231
333	52 244	52 257	52 270	52 284	52 297	52 310	52 323	52 336	52 349	52 362
334	52 375	52 388	52 401	52 414	52 427	52 440	52 453	52 466	52 479	52 492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
336	52 634	52 647	52 660	52 673	52 686	52 699	52 711	52 724	52 737	52 750
337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
338	52 892	52 905	52 917	52 930	52 943	52 956	52 969	52 982	52 994	53 007
339	53 020	53 033	53 046	53 058	53 071	53 084	53 097	53 110	53 122	53 135
340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
346	53 908	53 920	53 933	53 945	53 958	53 970	53 983	53 995	54 008	54 020
347	54 033	54 045	54 058	54 070	54 083	54 095	54 108	54 120	54 133	54 145
348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
349	54 283	54 295	54 307	54 320	54 332	54 345	54 357	54 370	54 382	54 394
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
351	54 531	54 543	54 555	54 568	54 580	54 593	54 605	54 617	54 630	54 642
352	54 654	54 667	54 679	54 691	54 704	54 716	54 728	54 741	54 753	54 765
353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
354	54 900	54 913	54 925	54 937	54 949	54 962	54 974	54 986	54 998	55 011
355	55 023	55 035	55 047	55 060	55 072	55 084	55 096	55 108	55 121	55 133
356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
358	55 388	55 400	55 413	55 425	55 437	55 449	55 461	55 473	55 485	55 497
359	55 509	55 522	55 534	55 546	55 558	55 570	55 582	55 594	55 606	55 618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
361	55 751	55 763	55 775	55 787	55 799	55 811	55 823	55 835	55 847	55 859
362	55 871	55 883	55 895	55 907	55 919	55 931	55 943	55 955	55 967	55 979
363	55 991	56 003	56 015	56 027	56 038	56 050	56 062	56 074	56 086	56 098
364	56 110	56 122	56 134	56 146	56 158	56 170	56 182	56 194	56 205	56 217
365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
369	56 703	56 714	56 726	56 738	56 750	56 761	56 773	56 785	56 797	56 808

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
370	56 820	56 832	56 844	56 855	56 867	56 879	56 891	56 902	56 914	56 926
371	56 937	56 949	56 961	56 972	56 984	56 996	57 008	57 019	57 031	57 043
372	57 054	57 066	57 078	57 089	57 101	57 113	57 124	57 136	57 148	57 159
373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
374	57 287	57 299	57 310	57 322	57 334	57 345	57 357	57 368	57 380	57 392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	57 519	57 530	57 542	57 553	57 565	57 576	57 588	57 600	57 611	57 623
377	57 634	57 646	57 657	57 669	57 680	57 692	57 703	57 715	57 726	57 738
378	57 749	57 761	57 772	57 784	57 795	57 807	57 818	57 830	57 841	57 852
379	57 864	57 875	57 887	57 898	57 910	57 921	57 933	57 944	57 955	57 967
380	57 978	57 990	58 001	58 013	58 024	58 035	58 047	58 058	58 070	58 081
381	58 092	58 104	58 115	58 127	58 138	58 149	58 161	58 172	58 184	58 195
382	58 206	58 218	58 229	58 240	58 252	58 263	58 274	58 286	58 297	58 309
383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
384	58 433	58 444	58 456	58 467	58 478	58 490	58 501	58 512	58 524	58 535
385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	58 659	58 670	58 681	58 692	58 704	58 715	58 726	58 737	58 749	58 760
387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
388	58 883	58 894	58 906	58 917	58 928	58 939	58 950	58 961	58 973	58 984
389	58 995	59 006	59 017	59 028	59 040	59 051	59 062	59 073	59 084	59 095
390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
397	59 879	59 890	59 901	59 912	59 923	59 934	59 945	59 956	59 966	59 977
398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	60 119	60 130	60 141	60 152	60 163	60 173	60 184	60 195
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
401	60 314	60 325	60 336	60 347	60 358	60 369	60 379	60 390	60 401	60 412
402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 885	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
408	61 066	61 077	61 087	61 098	61 109	61 119	61 130	61 140	61 151	61 162
409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
413	61 595	61 606	61 616	61 627	61 637	61 648	61 658	61 669	61 679	61 690
414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	62 941	62 951	62 961	62 972	62 982	62 992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	63 104	63 114	63 124	63 134
428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
429	63 246	63 256	63 266	63 276	63 286	63 296	63 306	63 317	63 327	63 337
430	63 347	63 357	63 367	63 377	63 387	63 397	63 407	63 417	63 428	63 438
431	63 448	63 458	63 468	63 478	63 488	63 498	63 508	63 518	63 528	63 538
432	63 548	63 558	63 568	64 579	63 589	63 599	63 609	63 619	63 629	63 639
433	63 649	63 659	63 669	63 679	63 689	63 699	63 709	63 719	63 729	63 739
434	63 749	63 759	63 769	63 779	63 789	63 799	63 809	63 819	63 829	63 839
435	63 849	63 859	63 869	63 879	63 889	63 899	63 909	63 919	63 929	63 939
436	63 949	63 959	63 969	63 979	63 988	63 998	64 008	64 018	64 028	64 038
437	64 048	64 058	64 068	64 078	64 088	64 098	64 108	64 118	64 128	64 137
438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
439	64 246	64 256	64 266	64 276	64 286	64 296	64 306	64 316	64 326	64 335
440	64 345	64 355	64 365	64 375	64 385	64 395	64 404	64 414	64 424	64 434
441	64 444	64 454	64 464	64 473	64 483	64 493	64 503	64 513	64 523	64 532
442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
444	64 738	64 748	64 758	64 768	64 777	64 787	64 797	64 807	64 816	64 826
445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
446	64 933	64 943	64 953	64 963	64 972	64 982	64 992	65 002	65 011	65 021
447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	65 099	65 108	65 118
448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 215
449	65 225	65 234	65 244	65 254	65 263	65 273	65 283	65 292	65 302	65 312
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
451	65 418	65 427	65 437	65 447	65 456	65 466	65 475	65 485	65 495	65 504
452	65 514	65 523	65 533	65 543	65 552	65 562	65 571	65 581	65 591	65 600
453	65 610	65 619	65 629	65 639	65 648	65 658	65 667	65 677	65 686	65 696
454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
458	66 087	66 096	66 106	66 115	66 124	66 134	66 143	66 153	66 162	66 172
459	66 181	66 191	66 200	66 210	66 219	66 229	66 238	66 247	66 257	66 266

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
460	66 276	66 285	66 295	66 304	66 314	66 323	66 332	66 342	66 351	66 361
461	66 370	66 380	66 389	66 398	66 408	66 417	66 427	66 436	66 445	66 455
462	66 464	66 474	66 483	66 492	66 502	66 511	66 521	66 530	66 539	66 549
463	66 558	66 567	66 577	66 586	66 596	66 605	66 614	66 624	66 633	66 642
464	66 652	66 661	66 671	66 680	66 689	66 699	66 708	66 717	66 727	66 736
465	66 745	66 755	66 764	66 773	66 783	66 792	66 801	66 811	66 820	66 829
466	66 839	66 848	66 857	66 867	66 876	66 885	66 894	66 904	66 913	66 922
467	66 932	66 941	66 950	66 960	66 969	66 978	66 987	66 997	67 006	67 015
468	67 025	67 034	67 043	67 052	67 062	67 071	67 080	67 089	67 099	67 108
469	67 117	67 127	67 136	67 145	67 154	67 164	67 173	67 182	67 191	67 201
470	67 210	67 219	67 228	67 237	67 247	67 256	67 265	67 274	67 284	67 293
471	67 302	67 311	67 321	67 330	67 339	67 348	67 357	67 367	67 376	67 385
472	67 394	67 403	67 413	67 422	67 431	67 440	67 449	67 459	67 468	67 477
473	67 486	67 495	67 504	67 514	67 523	67 532	67 541	67 550	67 560	67 569
474	67 578	67 587	67 596	67 605	67 614	67 624	67 633	67 642	67 651	67 660
475	67 669	67 679	67 688	67 697	67 706	67 715	67 724	67 733	67 742	67 752
476	67 761	67 770	67 779	67 788	67 797	67 806	67 815	67 825	67 834	67 843
477	67 852	67 861	67 870	67 879	67 888	67 897	67 906	67 916	67 925	67 934
478	67 943	67 952	67 961	67 970	67 979	67 988	67 997	68 006	68 015	68 024
479	68 034	68 043	68 052	68 061	68 070	68 079	68 088	68 097	68 106	68 115
480	68 124	68 133	68 142	68 151	68 160	68 169	68 178	68 187	68 196	68 205
481	68 215	68 224	68 233	68 242	68 251	68 260	68 269	68 278	68 287	68 296
482	68 305	68 314	68 323	68 332	68 341	68 350	68 359	68 368	68 377	68 386
483	68 395	68 404	68 413	68 422	68 431	68 440	68 449	68 458	68 467	68 476
484	68 485	68 494	68 502	68 511	68 520	68 529	68 538	68 547	68 556	68 565
485	68 574	68 583	68 592	68 601	68 610	68 619	68 628	68 637	68 646	68 655
486	68 664	68 673	68 681	68 690	68 699	68 708	68 717	68 726	68 735	68 744
487	68 753	68 762	68 771	68 780	68 789	68 797	68 806	68 815	68 824	68 833
488	68 842	68 851	68 860	68 869	68 878	68 886	68 895	68 904	68 913	68 922
489	68 931	68 940	68 949	68 958	68 966	68 975	68 984	68 993	69 002	69 011
490	69 020	69 028	69 037	69 046	69 055	69 064	69 073	69 082	69 090	69 099
491	69 108	69 117	69 126	69 135	69 144	69 152	69 161	69 170	69 179	69 188
492	69 197	69 205	69 214	69 223	69 232	69 241	69 249	69 258	69 267	69 276
493	69 285	69 294	69 302	69 311	69 320	69 329	69 338	69 346	69 355	69 364
494	69 373	69 381	69 390	69 399	69 408	69 417	69 425	69 434	69 443	69 452
495	69 461	69 469	69 478	69 487	69 496	69 504	69 513	69 522	69 531	69 539
496	69 548	69 557	69 566	69 574	69 583	69 592	69 601	69 609	69 618	69 627
497	69 636	69 644	69 653	69 662	69 671	69 679	69 688	69 697	69 705	69 714
498	69 723	69 732	69 740	69 749	69 758	69 767	69 775	69 784	69 793	69 801
499	69 810	69 819	69 827	69 836	69 845	69 854	69 862	69 871	69 880	69 888
500	69 897	69 906	69 914	69 923	69 932	69 940	69 949	69 958	69 966	69 975
501	69 984	69 992	70 001	70 010	70 018	70 027	70 036	70 044	70 053	70 062
502	70 070	70 079	70 088	70 096	70 105	70 114	70 122	70 131	70 140	70 148
503	70 157	70 165	70 174	70 183	70 191	70 200	70 209	70 217	70 226	70 234
504	70 243	70 252	70 260	70 269	70 278	70 286	70 295	70 303	70 312	70 321

COMMON LOGARITHMS OF NUMBERS
(Continued)

N	0	1	2	3	4	5	6	7	8	9
505	70 329	70 338	70 346	70 355	70 364	70 372	70 381	70 389	70 398	70 406
506	70 415	70 424	70 432	70 441	70 449	70 458	70 467	70 475	70 484	70 492
507	70 501	70 509	70 518	70 526	70 535	70 544	70 552	70 561	70 569	70 578
508	70 586	70 595	70 603	70 612	70 621	70 629	70 638	70 646	70 655	70 663
509	70 672	70 680	70 689	70 697	70 706	70 714	70 723	70 731	70 740	70 749
510	70 757	70 766	70 774	70 783	70 791	70 800	70 808	70 817	70 825	70 834
511	70 842	70 851	70 859	70 868	70 876	70 885	70 893	70 902	70 910	70 919
512	70 927	70 935	70 944	70 952	70 961	70 969	70 978	70 986	70 995	71 003
513	71 012	71 020	71 029	71 037	71 046	71 054	71 063	71 071	71 079	71 088
514	71 096	71 105	71 113	71 122	71 130	71 139	71 147	71 155	71 164	71 172
515	71 181	71 189	71 198	71 206	71 214	71 223	71 231	71 240	71 248	71 257
516	71 265	71 273	71 282	71 290	71 299	71 307	71 315	71 324	71 332	71 341
517	71 349	71 357	71 366	71 374	71 383	71 391	71 399	71 408	71 416	71 425
518	71 433	71 441	71 450	71 458	71 466	71 475	71 483	71 492	71 500	71 508
519	71 517	71 525	71 533	71 542	71 550	71 559	71 567	71 575	71 584	71 592
520	71 600	71 609	71 617	71 625	71 634	71 642	71 650	71 659	71 667	71 675
521	71 684	71 692	71 700	71 709	71 717	71 725	71 734	71 742	71 750	71 759
522	71 767	71 775	71 784	71 792	71 800	71 809	71 817	71 825	71 834	71 842
523	71 850	71 858	71 867	71 875	71 883	71 892	71 900	71 908	71 917	71 925
524	71 933	71 941	71 950	71 958	71 966	71 975	71 983	71 991	71 999	72 008
525	72 016	72 024	72 032	72 041	72 049	72 057	72 066	72 074	72 082	72 090
526	72 099	72 107	72 115	72 123	72 132	72 140	72 148	72 156	72 165	72 173
527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
529	72 346	72 354	72 362	72 370	72 378	72 387	72 395	72 403	72 411	72 419
530	72 428	72 436	72 444	72 452	72 460	72 469	72 477	72 485	72 493	72 501
531	72 509	72 518	72 526	72 534	72 542	72 550	72 558	72 567	72 575	72 583
532	72 591	72 599	72 607	72 616	72 624	72 632	72 640	72 648	72 656	72 665
533	72 673	72 681	72 689	72 697	72 705	72 713	72 722	72 730	72 738	72 746
534	72 754	72 762	72 770	72 779	72 787	72 795	72 803	72 811	72 819	72 827
535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
536	72 916	72 925	72 933	72 941	72 949	72 957	72 965	72 973	72 981	72 989
537	72 997	73 006	73 014	73 022	73 030	73 038	73 046	73 054	73 062	73 070
538	73 078	73 086	73 094	73 102	73 111	73 119	73 127	73 135	73 143	73 151
539	73 159	73 167	73 175	73 183	73 191	73 199	73 207	73 215	73 223	73 231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	73 320	73 328	73 336	73 344	73 352	73 360	73 368	73 376	73 384	73 392
542	73 400	73 408	73 416	73 424	73 432	73 440	73 448	73 456	73 464	73 472
543	73 480	73 488	73 496	73 504	73 512	73 520	73 528	73 536	73 544	73 552
544	73 560	73 568	73 576	73 584	73 592	73 600	73 608	73 616	73 624	73 632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
546	73 719	73 727	73 735	73 743	73 751	73 759	73 767	73 775	73 783	73 791
547	73 799	73 807	73 815	73 823	73 830	73 838	73 846	73 854	73 862	73 870
548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
549	73 957	73 965	73 973	73 981	73 989	73 997	74 005	74 013	74 020	74 028

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	75 282	75 289	75 297	75 305	75 312	75 320	75 328	75 335	75 343	75 351
567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
570	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
574	75 891	75 899	75 906	75 914	75 921	75 929	75 937	75 944	75 952	75 959
575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
580	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
585	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
589	77 012	77 019	77 026	77 034	77 041	77 048	77 056	77 063	77 070	77 078
590	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
598	77 670	77 677	77 685	77 692	77 699	77 706	77 714	77 721	77 728	77 735
599	77 743	77 750	77 757	77 764	77 772	77 779	77 786	77 793	77 801	77 808
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	77 887	77 895	77 902	77 909	77 916	77 924	77 931	77 938	77 945	77 952
602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
609	78 462	78 469	78 476	78 483	78 490	78 497	78 504	78 512	78 519	78 526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	80 072	80 079	80 085	80 092	80 099	80 106	80 113	80 120	80 127	80 134
633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	48 491	84 497	84 504
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 830	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376

COMMON LOGARITHMS OF NUMBERS
(Continued)

N	0	1	2	3	4	5	6	7	8	9
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697

COMMON LOGARITHMS OF NUMBERS
(Continued)

N	0	1	2	3	4	5	6	7	8	9
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996

COMMON LOGARITHMS OF NUMBERS
(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
872	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039

NATURAL LOGARITHMS OF NUMBERS FROM 1 TO 10 (Base *e*)

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0099	0.0198	0.0296	0.0392	0.0488	0.0583	0.0677	0.0770	0.0862
1.1	0.0953	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1740
1.2	0.1823	0.1906	0.1989	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4383	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6258	0.6313	0.6366
1.9	0.6419	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	0.6932	0.6981	0.7031	0.7080	0.7130	0.7178	0.7227	0.7276	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7885	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8373	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9001	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9820	0.9858	0.9895
2.7	0.9933	0.9970	1.0006	1.0043	1.0080	1.0116	1.0152	1.0189	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0681	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1970	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2670	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5603	1.5624	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5749	1.5769	1.5790	1.5810	1.5831	1.5852	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6545	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6753	1.6771	1.6790	1.6808	1.6827	1.6846
5.4	1.6864	1.6883	1.6901	1.6919	1.6938	1.6956	1.6975	1.6993	1.7011	1.7029

NATURAL LOGARITHMS OF NUMBERS
(Continued)

N	0	1	2	3	4	5	6	7	8	9
5.5	1.7048	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7491	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7682	1.7699	1.7716	1.7733
5.9	1.7750	1.7767	1.7783	1.7800	1.7817	1.7834	1.7851	1.7868	1.7884	1.7901
6.0	1.7918	1.7934	1.7951	1.7968	1.7984	1.8001	1.8017	1.8034	1.8050	1.8067
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8246	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8406	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9243	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9431	1.9445
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9545	1.9559	1.9573	1.9587
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0216	2.0229	2.0242	2.0255	2.0268
7.6	2.0282	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0669	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0882	2.0894	2.0906
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1151
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1259	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1354	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1816	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2149	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2365	2.2375	2.2386	2.2397
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016

NATURAL LOGARITHMS (EACH INCREASED BY 10) OF NUMBERS FROM 0.00 TO 0.99

No.	0	1	2	3	4	5	6	7	8	9
0.0	....	5.395	6.088	6.493	6.781	7.004	7.187	7.341	7.474	7.592
0.1	7.697	7.793	7.880	7.960	8.034	8.103	8.167	8.228	8.285	8.339
0.2	8.391	8.439	8.486	8.530	8.573	8.614	8.653	8.691	8.727	8.762
0.3	8.796	8.829	8.861	8.891	8.921	8.950	8.978	9.006	9.032	9.058
0.4	9.084	9.108	9.132	9.156	9.179	9.201	9.223	9.245	9.266	9.287
0.5	9.307	9.327	9.346	9.365	9.384	9.402	9.420	9.438	9.455	9.472
0.6	9.489	9.506	9.522	9.538	9.554	9.569	9.584	9.600	9.614	9.629
0.7	9.643	9.658	9.671	9.685	9.699	9.712	9.726	9.739	9.752	9.764
0.8	9.777	9.789	9.802	9.814	9.826	9.837	9.849	9.861	9.872	9.883
0.9	9.895	9.906	9.917	9.927	9.938	9.949	9.959	9.970	9.980	9.990

NATURAL LOGARITHMS OF WHOLE NUMBERS FROM 10 TO 209

No.	0	1	2	3	4	5	6	7	8	9
1	2.303	2.398	2.485	2.565	2.639	2.708	2.773	2.833	2.890	2.944
2	2.996	3.045	3.091	3.136	3.178	3.219	3.258	3.296	3.332	3.367
3	3.401	3.434	3.466	3.497	3.526	3.555	3.584	3.611	3.638	3.664
4	3.689	3.714	3.738	3.761	3.784	3.807	3.829	3.850	3.871	3.892
5	3.912	3.932	3.951	3.970	3.989	4.007	4.025	4.043	4.060	4.078
6	4.094	4.111	4.127	4.143	4.159	4.174	4.190	4.205	4.220	4.234
7	4.249	4.263	4.277	4.291	4.304	4.318	4.331	4.344	4.357	4.369
8	4.382	4.394	4.407	4.419	4.431	4.443	4.454	4.466	4.477	4.489
9	4.500	4.511	4.522	4.533	4.543	4.554	4.564	4.575	4.585	4.595
10	4.605	4.615	4.625	4.635	4.644	4.654	4.663	4.673	4.682	4.691
11	4.701	4.710	4.719	4.727	4.736	4.745	4.754	4.762	4.771	4.779
12	4.788	4.796	4.804	4.812	4.820	4.828	4.836	4.844	4.852	4.860
13	4.868	4.875	4.883	4.890	4.898	4.905	4.913	4.920	4.927	4.935
14	4.942	4.949	4.956	4.963	4.970	4.977	4.984	4.990	4.997	5.004
15	5.011	5.017	5.024	5.030	5.037	5.043	5.050	5.056	5.063	5.069
16	5.075	5.081	5.088	5.094	5.100	5.106	5.112	5.118	5.124	5.130
17	5.136	5.142	5.148	5.153	5.159	5.165	5.171	5.176	5.182	5.187
18	5.193	5.199	5.204	5.210	5.215	5.220	5.226	5.231	5.236	5.242
19	5.247	5.252	5.258	5.263	5.268	5.273	5.278	5.283	5.288	5.293
20	5.298	5.303	5.308	5.313	5.318	5.323	5.328	5.333	5.338	5.342

THE EXPONENTIAL e^x

For values of x from **0.000** to **0.099**

x	0	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	1.0000	1.0010	1.0020	1.0030	1.0040	1.0050	1.0060	1.0070	1.0080	1.0090
.01	1.0101	1.0111	1.0121	1.0131	1.0141	1.0151	1.0161	1.0171	1.0182	1.0191
.02	1.0202	1.0212	1.0222	1.0233	1.0243	1.0253	1.0263	1.0274	1.0284	1.0294
.03	1.0305	1.0315	1.0325	1.0336	1.0346	1.0356	1.0367	1.0377	1.0387	1.0398
.04	1.0408	1.0419	1.0429	1.0439	1.0450	1.0460	1.0471	1.0481	1.0492	1.0502
.05	1.0513	1.0523	1.0534	1.0544	1.0555	1.0565	1.0576	1.0587	1.0597	1.0608
.06	1.0618	1.0629	1.0640	1.0650	1.0661	1.0672	1.0682	1.0693	1.0704	1.0714
.07	1.0725	1.0736	1.0747	1.0757	1.0768	1.0779	1.0790	1.0800	1.0811	1.0822
.08	1.0833	1.0844	1.0855	1.0865	1.0876	1.0887	1.0898	1.0909	1.0920	1.0931
.09	1.0942	1.0953	1.0964	1.0975	1.0986	1.0997	1.1008	1.1019	1.1030	1.1041

For values of x from **0.10** to **2.99**

x	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.1	1.1052	1.1163	1.1275	1.1388	1.1503	1.1618	1.1735	1.1853	1.1972	1.2092
0.2	1.2214	1.2337	1.2461	1.2586	1.2712	1.2840	1.2969	1.3100	1.3231	1.3364
0.3	1.3499	1.3634	1.3771	1.3910	1.4049	1.4191	1.4333	1.4477	1.4623	1.4770
0.4	1.4918	1.5068	1.5220	1.5373	1.5527	1.5683	1.5841	1.6000	1.6161	1.6323
0.5	1.6487	1.6653	1.6820	1.6989	1.7160	1.7333	1.7507	1.7683	1.7860	1.8040
0.6	1.8221	1.8404	1.8589	1.8776	1.8965	1.9155	1.9348	1.9542	1.9739	1.9937
0.7	2.0138	2.0340	2.0544	2.0751	2.0959	2.1170	2.1383	2.1598	2.1815	2.2034
0.8	2.2255	2.2479	2.2705	2.2933	2.3164	2.3396	2.3632	2.3869	2.4109	2.4351
0.9	2.4596	2.4843	2.5093	2.5345	2.5600	2.5857	2.6117	2.6379	2.6645	2.6912
1.0	2.7183	2.7456	2.7732	2.8011	2.8292	2.8577	2.8864	2.9154	2.9447	2.9743
1.1	3.0042	3.0344	3.0649	3.0957	3.1268	3.1582	3.1899	3.2220	3.2544	3.2871
1.2	3.3201	3.3535	3.3872	3.4212	3.4556	3.4903	3.5254	3.5609	3.5966	3.6328
1.3	3.6693	3.7062	3.7434	3.7810	3.8190	3.8574	3.8962	3.9354	3.9749	4.0149
1.4	4.0552	4.0960	4.1371	4.1787	4.2207	4.2631	4.3060	4.3492	4.3929	4.4371
1.5	4.4817	4.5267	4.5722	4.6182	4.6646	4.7115	4.7588	4.8066	4.8550	4.9037
1.6	4.9530	5.0028	5.0531	5.1039	5.1552	5.2070	5.2593	5.3122	5.3656	5.4195
1.7	5.4739	5.5290	5.5845	5.6407	5.6973	5.7546	5.8124	5.8709	5.9299	5.9895
1.8	6.0496	6.1104	6.1719	6.2339	6.2965	6.3598	6.4237	6.4883	6.5535	6.6194
1.9	6.6859	6.7531	6.8210	6.8895	6.9588	7.0287	7.0993	7.1707	7.2427	7.3155
2.0	7.3891	7.4633	7.5383	7.6141	7.6906	7.7679	7.8460	7.9248	8.0045	8.0849
2.1	8.1662	8.2482	8.3311	8.4149	8.4994	8.5849	8.6711	8.7583	8.8463	8.9352
2.2	9.0250	9.1157	9.2073	9.2999	9.3933	9.4877	9.5831	9.6794	9.7767	9.8749
2.3	9.9742	10.074	10.176	10.278	10.381	10.486	10.591	10.697	10.805	10.913
2.4	11.023	11.134	11.246	11.359	11.473	11.588	11.705	11.822	11.941	12.061
2.3	12.182	12.305	12.429	12.554	12.680	12.807	12.936	13.066	13.197	13.330
2.6	13.464	13.599	13.736	13.874	14.013	14.154	14.296	14.440	14.585	14.732
2.7	14.880	15.029	15.180	15.333	15.487	15.643	15.800	15.959	16.119	16.281
2.8	16.445	16.610	16.777	16.945	17.116	17.288	17.462	17.637	17.814	17.993
2.9	18.174	18.357	18.541	18.728	18.916	19.106	19.298	19.492	19.688	19.886

For values of x from **3.0** to **8.9**

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
3	20.086	22.198	24.533	27.113	29.964	33.115	36.598	40.447	44.701	49.402
4	54.598	60.340	66.686	73.700	81.451	90.017	99.484	109.95	121.51	134.29
5	148.41	164.02	181.27	200.34	221.41	244.69	270.43	298.87	330.30	365.04
6	403.43	445.86	492.75	544.57	601.85	665.14	735.10	812.41	897.85	992.27
7	1096.6	1212.0	1339.4	1480.3	1636.0	1808.0	1998.2	2208.3	2440.6	2697.3
8	2981.0	3294.5	3641.0	4023.9	4447.1	4914.8	5431.7	6002.9	6634.2	7332.0

THE EXPONENTIAL e^{-x}

For values of x from **0.000** to **0.099.**

x	0	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	1.0000	.9990	.9980	.9970	.9960	.9950	.9940	.9930	.9920	.9910
.01	.9900	.9891	.9881	.9871	.9861	.9851	.9841	.9831	.9822	.9812
.02	.9802	.9792	.9782	.9773	.9763	.9753	.9743	.9734	.9724	.9714
.03	.9704	.9695	.9685	.9675	.9666	.9656	.9646	.9637	.9627	.9618
.04	.9608	.9598	.9589	.9579	.9570	.9560	.9550	.9541	.9531	.9522
.05	.9512	.9502	.9493	.9484	.9474	.9465	.9455	.9446	.9436	.9427
.06	.9418	.9408	.9399	.9389	.9380	.9371	.9361	.9352	.9343	.9333
.07	.9324	.9315	.9305	.9296	.9287	.9277	.9268	.9259	.9250	.9240
.08	.9231	.9222	.9213	.9204	.9194	.9185	.9176	.9167	.9158	.9148
.09	.9139	.9130	.9121	.9112	.9103	.9094	.9085	.9076	.9066	.9057

For values of x from **0.10** to **2.99**

x	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.1	.9048	.8958	.8869	.8781	.8694	.8607	.8521	.8437	.8353	.8270
0.2	.8187	.8106	.8025	.7945	.7866	.7788	.7711	.7634	.7558	.7483
0.3	.7408	.7334	.7261	.7189	.7118	.7047	.6977	.6907	.6839	.6771
0.4	.6703	.6637	.6570	.6505	.6440	.6376	.6313	.6250	.6188	.6126
0.5	.6065	.6005	.5945	.5886	.5827	.5769	.5712	.5655	.5599	.5543
0.6	.5488	.5434	.5379	.5326	.5273	.5220	.5169	.5117	.5066	.5016
0.7	.4966	.4916	.4868	.4819	.4771	.4724	.4677	.4630	.4584	.4538
0.8	.4493	.4449	.4404	.4360	.4317	.4274	.4232	.4190	.4148	.4107
0.9	.4066	.4025	.3985	.3946	.3906	.3867	.3829	.3791	.3753	.3716
1.0	.3679	.3642	.3606	.3570	.3535	.3499	.3465	.3430	.3396	.3362
1.1	.3329	.3296	.3263	.3230	.3198	.3166	.3135	.3104	.3073	.3042
1.2	.3012	.2982	.2952	.2923	.2894	.2865	.2837	.2808	.2780	.2753
1.3	.2725	.2698	.2671	.2645	.2618	.2592	.2567	.2541	.2516	.2491
1.4	.2466	.2441	.2417	.2393	.2369	.2346	.2322	.2299	.2276	.2254
1.5	.2231	.2209	.2187	.2165	.2144	.2122	.2101	.2080	.2060	.2039
1.6	.2019	.1999	.1979	.1959	.1940	.1920	.1901	.1882	.1864	.1845
1.7	.1827	.1809	.1791	.1773	.1755	.1738	.1720	.1703	.1686	.1670
1.8	.1653	.1637	.1620	.1604	.1588	.1572	.1557	.1541	.1526	.1511
1.9	.1496	.1481	.1466	.1451	.1437	.1423	.1409	.1395	.1381	.1367
2.0	.1353	.1340	.1327	.1313	.1300	.1287	.1275	.1262	.1249	.1237
2.1	.1225	.1212	.1200	.1188	.1177	.1165	.1153	.1142	.1130	.1119
2.2	.1108	.1097	.1086	.1075	.1065	.1054	.1044	.1033	.1023	.1013
2.3	.1003	.0993	.0983	.0973	.0963	.0954	.0944	.0935	.0926	.0916
2.4	.0907	.0898	.0889	.0880	.0872	.0863	.0854	.0846	.0837	.0829
2.5	.0821	.0813	.0805	.0797	.0789	.0781	.0773	.0765	.0758	.0750
2.6	.0743	.0735	.0728	.0721	.0714	.0707	.0699	.0693	.0686	.0679
2.7	.0672	.0665	.0659	.0652	.0646	.0639	.0633	.0627	.0620	.0614
2.8	.0608	.0602	.0596	.0590	.0584	.0578	.0573	.0567	.0561	.0556
2.9	.0550	.0545	.0539	.0534	.0529	.0523	.0518	.0513	.0508	.0503

For values of x from **3.0** to **8.9**

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
3	.0498	.0450	.0408	.0368	.0334	.0302	.0273	.0247	.0224	.0202
4	.0183	.0166	.0150	.0136	.0123	.0111	.0101	.0091	.0082	.0074
5	.0067	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027
6	.0025	.0022	.0020	.0018	.0017	.0015	.0014	.0012	.0011	.0010
7	.0009	.0008	.0007	.0007	.0006	.0006	.0005	.0005	.0004	.0004
8	.0003	.0003	.0003	.0002	.0002	.0002	.0002	.0002	.0002	.0001

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS

Degrees	sin	cos	tan	cot	
0° 00′	−∞	10.0000	−∞	+∞	90° 00′
0° 10′	7.4637	9.9999	7.4637	2.5363	89° 50′
0° 20′	7.7648	9.9999	7.7648	2.2352	89° 40′
0° 30′	7.9408	9.9999	7.9409	2.0591	89° 30′
0° 40′	8.0658	9.9999	8.0658	1.9342	89° 20′
0° 50′	8.1627	9.9999	8.1627	1.8373	89° 10′
1° 00′	8.2419	9.9999	8.2419	1.7581	89° 00′
1° 10′	8.3088	9.9999	8.3089	1.6911	88° 50′
1° 20′	8.3668	9.9999	8.3669	1.6331	88° 40′
1° 30′	8.4179	9.9999	8.4181	1.5819	88° 30′
1° 40′	8.4637	9.9998	8.4638	1.5362	88° 20′
1° 50′	8.5050	9.9998	8.5053	1.4947	88° 10′
2° 00′	8.5428	9.9997	8.5431	1.4569	88° 00′
2° 10′	8.5776	9.9997	8.5779	1.4221	87° 50′
2° 20′	8.6097	9.9996	8.6101	1.3899	87° 40′
2° 30′	8.6397	9.9996	8.6401	1.3599	87° 30′
2° 40′	8.6677	9.9995	8.6682	1.3318	87° 20′
2° 50′	8.6940	9.9995	8.6945	1.3055	87° 10′
3° 00′	8.7188	9.9994	8.7194	1.2806	87° 00′
3° 10′	8.7423	9.9993	8.7429	1.2571	86° 50′
3° 20′	8.7645	9.9993	8.7652	1.2348	86° 40′
3° 30′	8.7857	9.9992	8.7865	1.2135	86° 30′
3° 40′	8.8059	9.9991	8.8067	1.1933	86° 20′
3° 50′	8.8251	9.9990	8.8261	1.1739	86° 10′
4° 00′	8.8436	9.9989	8.8446	1.1554	86° 00′
4° 10′	8.8613	9.9989	8.8624	1.1376	85° 50′
4° 20′	8.8783	9.9988	8.8795	1.1205	85° 40′
4° 30′	8.8946	9.9987	8.8960	1.1040	85° 30′
4° 40′	8.9104	9.9986	8.9118	1.0882	85° 20′
4° 50′	8.9256	9.9985	8.9272	1.0728	85° 10′
5° 00′	8.9403	9.9983	8.9420	1.0580	85° 00′
5° 10′	8.9545	9.9982	8.9563	1.0437	84° 50′
5° 20′	8.9682	9.9981	8.9701	1.0299	84° 40′
5° 30′	8.9816	9.9980	8.9836	1.0164	84° 30′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
5° 40′	8.9945	9.9979	8.9966	1.0034	84° 20′
5° 50′	9.0070	9.9977	9.0093	0.9907	84° 10′
6° 00′	9.0192	9.9976	9.0216	0.9784	84° 00′
6° 10′	9.0311	9.9975	9.0336	0.9664	83° 50′
6° 20′	9.0426	9.9973	9.0453	0.9547	83° 40′
6° 30′	9.0539	9.9972	9.0567	0.9433	83° 30′
6° 40′	9.0648	9.9971	9.0678	0.9322	83° 20′
6° 50′	9.0755	9.9969	9.0786	0.9214	83° 10′
7° 00′	9.0859	9.9968	9.0891	0.9109	83° 00′
7° 10′	9.0961	9.9966	9.0995	0.9005	82° 50′
7° 20′	9.1060	9.9964	9.1096	0.8904	82° 40′
7° 30′	9.1157	9.9963	9.1194	0.8806	82° 30′
7° 40′	9.1252	9.9961	9.1291	0.8709	82° 20′
7° 50′	9.1345	9.9959	9.1385	0.8615	82° 10′
8° 00′	9.1436	9.9958	9.1478	0.8522	82° 00′
8° 10′	9.1525	9.9956	9.1569	0.8431	81° 50′
8° 20′	9.1612	9.9954	9.1658	0.8342	81° 40′
8° 30′	9.1697	9.9952	9.1745	0.8255	81° 30′
8° 40′	9.1781	9.9950	9.1831	0.8169	81° 20′
8° 50′	9.1863	9.9948	9.1915	0.8085	81° 10′
9° 00′	9.1943	9.9946	9.1997	0.8003	81° 00′
9° 10′	9.2022	9.9944	9.2078	0.7922	80° 50′
9° 20′	9.2100	9.9942	9.2158	0.7842	80° 40′
9° 30′	9.2176	9.9940	9.2236	0.7764	80° 30′
9° 40′	9.2251	9.9938	9.2313	0.7687	80° 20′
9° 50′	9.2324	9.9936	9.2389	0.7611	80° 10′
10° 00′	9.2397	9.9934	9.2463	0.7537	80° 00′
10° 10′	9.2468	9.9931	9.2536	0.7464	79° 50′
10° 20′	9.2538	9.9929	9.2609	0.7391	79° 40′
10° 30′	9.2606	9.9927	9.2680	0.7320	79° 30′
10° 40′	9.2674	9.9924	9.2750	0.7250	79° 20′
10° 50′	9.2740	9.9922	9.2819	0.7181	79° 10′
11° 00′	9.2806	9.9919	9.2887	0.7113	79° 00′
11° 10′	9.2870	9.9917	9.2953	0.7047	78° 50′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
11° 20′	9.2934	9.9914	9.3020	0.6980	78° 40′
11° 30′	9.2997	9.9912	9.3085	0.6915	78° 30′
11° 40′	9.3058	9.9909	9.3149	0.6851	78° 20′
11° 50′	9.3119	9.9907	9.3212	0.6788	78° 10′
12° 00′	9.3179	9.9904	9.3275	0.6725	78° 00′
12° 10′	9.3238	9.9901	9.3336	0.6664	77° 50′
12° 20′	9.3296	9.9899	9.3397	0.6603	77° 40′
12° 30′	9.3353	9.9896	9.3458	0.6542	77° 30′
12° 40′	9.3410	9.9893	9.3517	0.6483	77° 20′
12° 50′	9.3466	9.9890	9.3576	0.6424	77° 10′
13° 00′	9.3521	9.9887	9.3634	0.6366	77° 00′
13° 10′	9.3575	9.9884	9.3691	0.6309	76° 50′
13° 20′	9.3629	9.9881	9.3748	0.6252	76° 40′
13° 30′	9.3682	9.9878	9.3804	0.6196	76° 30′
13° 40′	9.3734	9.9875	9.3859	0.6141	76° 20′
13° 50′	9.3786	9.9872	9.3914	0.6086	76° 10′
14° 00′	9.3837	9.9869	9.3968	0.6032	76° 00′
14° 10′	9.3887	9.9866	9.4021	0.5979	75° 50′
14° 20′	9.3937	9.9863	9.4074	0.5926	75° 40′
14° 30′	9.3986	9.9859	9.4127	0.5873	75° 30′
14° 40′	9.4035	9.9856	9.4178	0.5822	75° 20′
14° 50′	9.4083	9.9853	9.4230	0.5770	75° 10′
15° 00′	9.4130	9.9849	9.4281	0.5719	75° 00′
15° 10′	9.4177	9.9846	9.4331	0.5669	74° 50′
15° 20′	9.4223	9.9843	9.4381	0.5619	74° 40′
15° 30′	9.4269	9.9839	9.4430	0.5570	74° 30′
15° 40′	9.4314	9.9836	9.4479	0.5521	74° 20′
15° 50′	9.4359	9.9832	9.4527	0.5473	74° 10′
16° 00′	9.4403	9.9828	9.4575	0.5425	74° 00′
16° 10′	9.4447	9.9825	9.4622	0.5378	73° 50′
16° 20′	9.4491	9.9821	9.4669	0.5331	73° 40′
16° 30′	9.4533	9.9817	9.4716	0.5284	73° 30′
16° 40′	9.4576	9.9814	9.4762	0.5238	73° 20′
16° 50′	9.4618	9.9810	9.4808	0.5192	73° 10′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
17° 00′	9.4659	9.9806	9.4853	0.5147	73° 00′
17° 10′	9.4700	9.9802	9.4898	0.5102	72° 50′
17° 20′	9.4741	9.9798	9.4943	0.5057	72° 40′
17° 30′	9.4781	9.9794	9.4987	0.5013	72° 30′
17° 40′	9.4821	9.9790	9.5031	0.4969	72° 20′
17° 50′	9.4861	9.9786	9.5075	0.4925	72° 10′
18° 00′	9.4900	9.9782	9.5118	0.4882	72° 00′
18° 10′	9.4939	9.9778	9.5161	0.4839	71° 50′
18° 20′	9.4977	9.9774	9.5203	0.4797	71° 40′
18° 30′	9.5015	9.9770	9.5245	0.4755	71° 30′
18° 40′	9.5052	9.9765	9.5287	0.4713	71° 20′
18° 50′	9.5090	9.9761	9.5329	0.4671	71° 10′
19° 00′	9.5126	9.9757	9.5370	0.4630	71° 00′
19° 10′	9.5163	9.9752	9.5411	0.4589	70° 50′
19° 20′	9.5199	9.9748	9.5451	0.4549	70° 40′
19° 30′	9.5235	9.9743	9.5491	0.4509	70° 30′
19° 40′	9.5270	9.9739	9.5531	0.4469	70° 20′
19° 50′	9.5306	9.9734	9.5571	0.4429	70° 10′
20° 00′	9.5341	9.9730	9.5611	0.4389	70° 00′
20° 10′	9.5375	9.9725	9.5650	0.4350	69° 50′
20° 20′	9.5409	9.9721	9.5689	0.4311	69° 40′
20° 30′	9.5443	9.9716	9.5727	0.4273	69° 30′
20° 40′	9.5477	9.9711	9.5766	0.4234	69° 20′
20° 50′	9.5510	9.9706	9.5804	0.4196	69° 10′
21° 00′	9.5543	9.9702	9.5842	0.4158	69° 00′
21° 10′	9.5576	9.9697	9.5879	0.4121	68° 50′
21° 20′	9.5609	9.9692	9.5917	0.4083	68° 40′
21° 30′	9.5641	9.9687	9.5954	0.4046	68° 30′
21° 40′	9.5673	9.9682	9.5991	0.4009	68° 20′
21° 50′	9.5704	9.9677	9.6028	0.3972	68° 10′
22° 00′	9.5736	9.9672	9.6064	0.3936	68° 00′
22° 10′	9.5767	9.9667	9.6100	0.3900	67° 50′
22° 20′	9.5798	9.9661	9.6136	0.3864	67° 40′
22° 30′	9.5828	9.9656	9.6172	0.3828	67° 30′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
22° 40′	9.5859	9.9651	9.6208	0.3792	67° 20′
22° 50′	9.5889	9.9646	9.6243	0.3757	67° 10′
23° 00′	9.5919	9.9640	9.6279	0.3721	67° 00′
23° 10′	9.5948	9.9635	9.6314	0.3686	66° 50′
23° 20′	9.5978	9.9629	9.6348	0.3652	66° 40′
23° 30′	9.6007	9.9624	9.6383	0.3617	66° 30′
23° 40′	9.6036	9.9618	9.6417	0.3583	66° 20′
23° 50′	9.6065	9.9613	9.6452	0.3548	66° 10′
24° 00′	9.6093	9.9607	9.6486	0.3514	66° 00′
24° 10′	9.6121	9.9602	9.6520	0.3480	65° 50′
24° 20′	9.6149	9.9596	9.6553	0.3447	65° 40′
24° 30′	9.6177	9.9590	9.6587	0.3413	65° 30′
24° 40′	9.6205	9.9584	9.6620	0.3380	65° 20′
24° 50′	9.6232	9.9579	9.6654	0.3346	65° 10′
25° 00′	9.6259	9.9573	9.6687	0.3313	65° 00′
25° 10′	9.6286	9.9567	9.6720	0.3280	64° 50′
25° 20′	9.6313	9.9561	9.6752	0.3248	64° 40′
25° 30′	9.6340	9.9555	9.6785	0.3215	64° 30′
25° 40′	9.6366	9.9549	9.6817	0.3183	64° 20′
25° 50′	9.6392	9.9543	9.6850	0.3150	64° 10′
26° 00′	9.6418	9.9537	9.6882	0.3118	64° 00′
26° 10′	9.6444	9.9530	9.6914	0.3086	63° 50′
26° 20′	9.6470	9.9524	9.6946	0.3054	63° 40′
26° 30′	9.6495	9.9518	9.6977	0.3023	63° 30′
26° 40′	9.6521	9.9512	9.7009	0.2991	63° 20′
26° 50′	9.6546	9.9505	9.7040	0.2960	63° 10′
27° 00′	9.6570	9.9499	9.7072	0.2928	63° 00′
27° 10′	9.6595	9.9492	9.7103	0.2897	62° 50′
27° 20′	9.6620	9.9486	9.7134	0.2866	62° 40′
27° 30′	9.6644	9.9479	9.7165	0.2835	62° 30′
27° 40′	9.6668	9.9473	9.7196	0.2804	62° 20′
27° 50′	9.6692	9.9466	9.7226	0.2774	62° 10′
28° 00′	9.6716	9.9459	9.7257	0.2743	62° 00′
28° 10′	9.6740	9.9453	9.7287	0.2713	61° 50′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
28° 20′	9.6763	9.9446	9.7317	0.2683	61° 40′
28° 30′	9.6787	9.9439	9.7348	0.2652	61° 30′
28° 40′	9.6810	9.9432	9.7378	0.2622	61° 20′
28° 50′	9.6833	9.9425	9.7408	0.2592	61° 10′
29° 00′	9.6856	9.9418	9.7438	0.2562	61° 00′
29° 10′	9.6878	9.9411	9.7467	0.2533	60° 50′
29° 20′	9.6901	9.9404	9.7497	0.2503	60° 40′
29° 30′	9.6923	9.9397	9.7526	0.2474	60° 30′
29° 40′	9.6946	9.9390	9.7556	0.2444	60° 20′
29° 50′	9.6968	9.9383	9.7585	0.2415	60° 10′
30° 00′	9.6990	9.9375	9.7614	0.2386	60° 00′
30° 10′	9.7012	9.9368	9.7644	0.2356	59° 50′
30° 20′	9.7033	9.9361	9.7673	0.2327	59° 40′
30° 30′	9.7055	9.9353	9.7701	0.2299	59° 30′
30° 40′	9.7076	9.9346	9.7730	0.2270	59° 20′
30° 50′	9.7097	9.9338	9.7759	0.2241	59° 10′
31° 00′	9.7118	9.9331	9.7788	0.2212	59° 00′
31° 10′	9.7139	9.9323	9.7816	0.2184	58° 50′
31° 20′	9.7160	9.9315	9.7845	0.2155	58° 40′
31° 30′	9.7181	9.9308	9.7873	0.2127	58° 30′
31° 40′	9.7201	9.9300	9.7902	0.2098	58° 20′
31° 50′	9.7222	9.9292	9.7930	0.2070	58° 10′
32° 00′	9.7242	9.9284	9.7958	0.2042	58° 00′
32° 10′	9.7262	9.9276	9.7986	0.2014	57° 50′
32° 20′	9.7282	9.9268	9.8014	0.1986	57° 40′
32° 30′	9.7302	9.9260	9.8042	0.1958	57° 30′
32° 40′	9.7322	9.9252	9.8070	0.1930	57° 20′
32° 50′	9.7342	9.9244	9.8097	0.1903	57° 10′
33° 00′	9.7361	9.9236	9.8125	0.1875	57° 00′
33° 10′	9.7380	9.9228	9.8153	0.1847	56° 50′
33° 20′	9.7400	9.9219	9.8180	0.1820	56° 40′
33° 30′	9.7419	9.9211	9.8208	0.1792	56° 30′
33° 40′	9.7438	9.9203	9.8235	0.1765	56° 20′
33° 50′	9.7457	9.9194	9.8263	0.1737	56° 10′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS. (*Continued*)

Degrees	sin	cos	tan	cot	
34° 00′	9.7476	9.9186	9.8290	0.1710	56° 00′
34° 10′	9.7494	9.9177	9.8317	0.1683	55° 50′
34° 20′	9.7513	9.9169	9.8344	0.1656	55° 40′
34° 30′	9.7531	9.9160	9.8371	0.1629	55° 30′
34° 40′	9.7550	9.9151	9.8398	0.1602	55° 20′
34° 50′	9.7568	9.9142	9.8425	0.1575	55° 10′
35° 00′	9.7586	9.9134	9.8452	0.1548	55° 00′
35° 10′	9.7604	9.9125	9.8479	0.1521	54° 50′
35° 20′	9.7622	9.9116	9.8506	0.1494	54° 40′
35° 30′	9.7640	9.9107	9.8533	0.1467	54° 30′
35° 40′	9.7657	9.9098	9.8559	0.1441	54° 20′
35° 50′	9.7675	9.9089	9.8586	0.1414	54° 10′
36° 00′	9.7692	9.9080	9.8613	0.1387	54° 00′
36° 10′	9.7710	9.9070	9.8639	0.1361	53° 50′
36° 20′	9.7727	9.9061	9.8666	0.1334	53° 40′
36° 30′	9.7744	9.9052	9.8692	0.1308	53° 30′
36° 40′	9.7761	9.9042	9.8718	0.1282	53° 20′
36° 50′	9.7778	9.9033	9.8745	0.1255	53° 10′
37° 00′	9.7795	9.9023	9.8771	0.1229	53° 00′
37° 10′	9.7811	9.9014	9.8797	0.1203	52° 50′
37° 20′	9.7828	9.9004	9.8824	0.1176	52° 40′
37° 30′	9.7844	9.8995	9.8850	0.1150	52° 30′
37° 40′	9.7861	9.8985	9.8876	0.1124	52° 20′
37° 50′	9.7877	9.8975	9.8902	0.1098	52° 10′
38° 00′	9.7893	9.8965	9.8928	0.1072	52° 00′
38° 10′	9.7910	9.8955	9.8954	0.1046	51° 50′
38° 20′	9.7926	9.8945	9.8980	0.1020	51° 40′
38° 30′	9.7941	9.8935	9.9006	0.0994	51° 30′
38° 40′	9.7957	9.8925	9.9032	0.0968	51° 20′
38° 50′	9.7973	9.8915	9.9058	0.0942	51° 10′
39° 00′	9.7989	9.8905	9.9084	0.0916	51° 00′
39° 10′	9.8004	9.8895	9.9110	0.0890	50° 50′
39° 20′	9.8020	9.8884	9.9135	0.0865	50° 40′
39° 30′	9.8035	9.8874	9.9161	0.0839	50° 30′
39° 40′	9.8050	9.8864	9.9187	0.0813	50° 20′
39° 50′	9.8066	9.8853	9.9212	0.0788	50° 10′
	cos	sin	cot	tan	Degrees

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
40° 00′	9.8081	9.8843	9.9238	0.0762	50° 00′
40° 10′	9.8096	9.8832	9.9264	0.0736	49° 50′
40° 20′	9.8111	9.8821	9.9289	0.0711	49° 40′
40° 30′	9.8125	9.8810	9.9315	0.0685	49° 30′
40° 40′	9.8140	9.8800	9.9341	0.0659	49° 20′
40° 50′	9.8155	9.8789	9.9366	0.0634	49° 10′
41° 00′	9.8169	9.8778	9.9392	0.0608	49° 00′
41° 10′	9.8184	9.8767	9.9417	0.0583	48° 50′
41° 20′	9.8198	9.8756	9.9443	0.0557	48° 40′
41° 30′	9.8213	9.8745	9.9468	0.0532	48° 30′
41° 40′	9.8227	9.8733	9.9494	0.0506	48° 20′
41° 50′	9.8241	9.8722	9.9519	0.0481	48° 10′
42° 00′	9.8255	9.8711	9.9544	0.0456	48° 00′
42° 10′	9.8269	9.8699	9.9570	0.0430	47° 50′
42° 20′	9.8283	9.8688	9.9595	0.0405	47° 40′
42° 30′	9.8297	9.8676	9.9621	0.0379	47° 30′
42° 40′	9.8311	9.8665	9.9646	0.0354	47° 20′
42° 50′	9.8324	9.8653	9.9671	0.0329	47° 10′
43° 00′	9.8338	9.8641	9.9697	0.0303	47° 00′
43° 10′	9.8351	9.8629	9.9722	0.0278	46° 50′
43° 20′	9.8365	9.8618	9.9747	0.0253	46° 40′
43° 30′	9.8378	9.8606	9.9772	0.0228	46° 30′
43° 40′	9.8391	9.8594	9.9798	0.0202	46° 20′
43° 50′	9.8405	9.8582	9.9823	0.0177	46° 10′
44° 00′	9.8418	9.8569	9.9848	0.0152	46° 00′
44° 10′	9.8431	9.8557	9.9874	0.0126	45° 50′
44° 20′	9.8444	9.8545	9.9899	0.0101	45° 40′
44° 30′	9.8457	9.8532	9.9924	0.0076	45° 30′
44° 40′	9.8469	9.8520	9.9949	0.0051	45° 20′
44° 50′	9.8482	9.8507	9.9975	0.0025	45° 10′
45° 00′	9.8495	9.8495	0.0000	0.0000	45° 00′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

Degrees	sin	cos	tan	cot	
0° 00′	.0000	1.0000	.0000	∞	90° 00′
0° 10′	.0029	1.0000	.0029	343.77	89° 50′
0° 20′	.0058	1.0000	.0058	171.89	89° 40′
0° 30′	.0087	1.0000	.0087	114.59	89° 30′
0° 40′	.0116	.9999	.0116	85.940	89° 20′
0° 50′	.0145	.9999	.0145	68.750	89° 10′
1° 00′	.0175	.9998	.0175	57.290	89° 00′
1° 10′	.0204	.9998	.0204	49.104	88° 50′
1° 20′	.0233	.9997	.0233	42.964	88° 40′
1° 30′	.0262	.9997	.0262	38.188	88° 30′
1° 40′	.0291	.9996	.0291	34.368	88° 20′
1° 50′	.0320	.9995	.0320	31.242	88° 10′
2° 00′	.0349	.9994	.0349	28.636	88° 00′
2° 10′	.0378	.9993	.0378	26.432	87° 50′
2° 20′	.0407	.9992	.0407	24.542	87° 40′
2° 30′	.0436	.9990	.0437	22.904	87° 30′
2° 40′	.0465	.9989	.0466	21.470	87° 20′
2° 50′	.0494	.9988	.0495	20.206	87° 10′
3° 00′	.0523	.9986	.0524	19.081	87° 00′
3° 10′	.0552	.9985	.0553	18.075	86° 50′
3° 20′	.0581	.9983	.0582	17.169	86° 40′
3° 30′	.0610	.9981	.0612	16.350	86° 30′
3° 40′	.0640	.9980	.0641	15.605	86° 20′
3° 50′	.0669	.9978	.0670	14.924	86° 10′
4° 00′	.0698	.9976	.0699	14.301	86° 00′
4° 10′	.0727	.9974	.0729	13.727	85° 50′
4° 20′	.0756	.9971	.0758	13.197	85° 40′
4° 30′	.0785	.9969	.0787	12.706	85° 30′
4° 40′	.0814	.9967	.0816	12.251	85° 20′
4° 50′	.0843	.9964	.0846	11.826	85° 10′
5° 00′	.0872	.9962	.0875	11.430	85° 00′
5° 10′	.0901	.9959	.0904	11.059	84° 50′
5° 20′	.0929	.9957	.0934	10.712	84° 40′
5° 30′	.0958	.9954	.0963	10.385	84° 30′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
5° 40′	.0987	.9951	.0992	10.078	84° 20′
5° 50′	.1016	.9948	.1022	9.7882	84° 10′
6° 00′	.1045	.9945	.1051	9.5144	84° 00′
6° 10′	.1074	.9942	.1080	9.2553	83° 50′
6° 20′	.1103	.9939	.1110	9.0098	83° 40′
6° 30′	.1132	.9936	.1139	8.7769	83° 30′
6° 40′	.1161	.9932	.1169	8.5555	83° 20′
6° 50′	.1190	.9929	.1198	8.3450	83° 10′
7° 00′	.1219	.9925	.1228	8.1443	83° 00′
7° 10′	.1248	.9922	.1257	7.9530	82° 50′
7° 20′	.1276	.9918	.1287	7.7704	82° 40′
7° 30′	.1305	.9914	.1317	7.5958	82° 30′
7° 40′	.1334	.9911	.1346	7.4287	82° 20′
7° 50′	.1363	.9907	.1376	7.2687	82° 10′
8° 00′	.1392	.9903	.1405	7.1154	82° 00′
8° 10′	.1421	.9899	.1435	6.9682	81° 50′
8° 20′	.1449	.9894	.1465	6.8269	81° 40′
8° 30′	.1478	.9890	.1495	6.6912	81° 30′
8° 40′	.1507	.9886	.1524	6.5606	81° 20′
8° 50′	.1536	.9881	.1554	6.4348	81° 10′
9° 00′	.1564	.9877	.1584	6.3138	81° 00′
9° 10′	.1593	.9872	.1614	6.1970	80° 50′
9° 20′	.1622	.9868	.1644	6.0844	80° 40′
9° 30′	.1650	.9863	.1673	5.9758	80° 30′
9° 40′	.1679	.9858	.1703	5.8708	80° 20′
9° 50′	.1708	.9853	.1733	5.7694	80° 10′
10° 00′	.1736	.9848	.1763	5.6713	80° 00′
10° 10′	.1765	.9843	.1793	5.5764	79° 50′
10° 20′	.1794	.9838	.1823	5.4845	79° 40′
10° 30′	.1822	.9833	.1853	5.3955	79° 30′
10° 40′	.1851	.9827	.1883	5.3093	79° 20′
10° 50′	.1880	.9822	.1914	5.2257	79° 10′
11° 00′	.1908	.9816	.1944	5.1446	79° 00′
11° 10′	.1937	.9811	.1974	5.0658	78° 50′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
11° 20′	.1965	.9805	.2004	4.9894	78° 40′
11° 30′	.1994	.9799	.2035	4.9152	78° 30′
11° 40′	.2022	.9793	.2065	4.8430	78° 20′
11° 50′	.2051	.9787	.2095	4.7729	78° 10′
12° 00′	.2079	.9781	.2126	4.7046	78° 00′
12° 10′	.2108	.9775	.2156	4.6382	77° 50′
12° 20′	.2136	.9769	.2186	4.5736	77° 40′
12° 30′	.2164	.9763	.2217	4.5107	77° 30′
12° 40′	.2193	.9757	.2247	4.4494	77° 20′
12° 50′	.2221	.9750	.2278	4.3897	77° 10′
13° 00′	.2250	.9744	.2309	4.3315	77° 00′
13° 10′	.2278	.9737	.2339	4.2747	76° 50′
13° 20′	.2306	.9730	.2370	4.2193	76° 40′
13° 30′	.2334	.9724	.2401	4.1653	76° 30′
13° 40′	.2363	.9717	.2432	4.1126	76° 20′
13° 50′	.2391	.9710	.2462	4.0611	76° 10′
14° 00′	.2419	.9703	.2493	4.0108	76° 00′
14° 10′	.2447	.9696	.2524	3.9617	75° 50′
14° 20′	.2476	.9689	.2555	3.9136	75° 40′
14° 30′	.2504	.9681	.2586	3.8667	75° 30′
14° 40′	.2532	.9674	.2617	3.8208	75° 20′
14° 50′	.2560	.9667	.2648	3.7760	75° 10′
15° 00′	.2588	.9659	.2679	3.7321	75° 00′
15° 10′	.2616	.9652	.2711	3.6891	74° 50′
15° 20′	.2644	.9644	.2742	3.6470	74° 40′
15° 30′	.2672	.9636	.2773	3.6059	74° 30′
15° 40′	.2700	.9628	.2805	3.5656	74° 20′
15° 50′	.2728	.9621	.2836	3.5261	74° 10′
16° 00′	.2756	.9613	.2867	3.4874	74° 00′
16° 10′	.2784	.9605	.2899	3.4495	73° 50′
16° 20′	.2812	.9596	.2931	3.4124	73° 40′
16° 30′	.2840	.9588	.2962	3.3759	73° 30′
16° 40′	.2868	.9580	.2994	3.3402	73° 20′
16° 50′	.2896	.9572	.3026	3.3052	73° 10′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
17° 00′	.2924	.9563	.3057	3.2709	73° 00′
17° 10′	.2952	.9555	.3089	3.2371	72° 50′
17° 20′	.2979	.9546	.3121	3.2041	72° 40′
17° 30′	.3007	.9537	.3153	3.1716	72° 30′
17° 40′	.3035	.9528	.3185	3.1397	72° 20′
17° 50′	.3062	.9520	.3217	3.1084	72° 10′
18° 00′	.3090	.9511	.3249	3.0777	72° 00′
18° 10′	.3118	.9502	.3281	3.0475	71° 50′
18° 20′	.3145	.9492	.3314	3.0178	71° 40′
18° 30′	.3173	.9483	.3346	2.9887	71° 30′
18° 40′	.3201	.9474	.3378	2.9600	71° 20′
18° 50′	.3228	.9465	.3411	2.9319	71° 10′
19° 00′	.3256	.9455	.3443	2.9042	71° 00′
19° 10′	.3283	.9446	.3476	2.8770	70° 50′
19° 20′	.3311	.9436	.3508	2.8502	70° 40′
19° 30′	.3338	.9426	.3541	2.8239	70° 30′
19° 40′	.3365	.9417	.3574	2.7980	70° 20′
19° 50′	.3393	.9407	.3607	2.7725	70° 10′
20° 00′	.3420	.9397	.3640	2.7475	70° 00′
20° 10′	.3448	.9387	.3673	2.7228	69° 50′
20° 20′	.3475	.9377	.3706	2.6985	69° 40′
20° 30′	.3502	.9367	.3739	2.6746	69° 30′
20° 40′	.3529	.9356	.3772	2.6511	69° 20′
20° 50′	.3557	.9346	.3805	2.6279	69° 10′
21° 00′	.3584	.9336	.3839	2.6051	69° 00′
21° 10′	.3611	.9325	.3872	2.5826	68° 50′
21° 20′	.3638	.9315	.3906	2.5605	68° 40′
21° 30′	.3665	.9304	.3939	2.5386	68° 30′
21° 40′	.3692	.9293	.3973	2.5172	68° 20′
21° 50′	.3719	.9283	.4006	2.4960	68° 10′
22° 00′	.3746	.9272	.4040	2.4751	68° 00′
22° 10′	.3773	.9261	.4074	2.4545	67° 50′
22° 20′	.3800	.9250	.4108	2.4342	67° 40′
22° 30′	.3827	.9239	.4142	2.4142	67° 30′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
22° 40′	.3854	.9228	.4176	2.3945	67° 20′
22° 50′	.3881	.9216	.4210	2.3750	67° 10′
23° 00′	.3907	.9205	.4245	2.3559	67° 00′
23° 10′	.3934	.9194	.4279	2.3369	66° 50′
23° 20′	.3961	.9182	.4314	2.3183	66° 40′
23° 30′	.3987	.9171	.4348	2.2998	66° 30′
23° 40′	.4014	.9159	.4383	2.2817	66° 20′
23° 50′	.4041	.9147	.4417	2.2637	66° 10′
24° 00′	.4067	.9135	.4452	2.2460	66° 00′
24° 10′	.4094	.9124	.4487	2.2286	65° 50′
24° 20′	.4120	.9112	.4522	2.2113	65° 40′
24° 30′	.4147	.9100	.4557	2.1943	65° 30′
24° 40′	.4173	.9088	.4592	2.1775	65° 20′
24° 50′	.4200	.9075	.4628	2.1609	65° 10′
25° 00′	.4226	.9063	.4663	2.1445	65° 00′
25° 10′	.4253	.9051	.4699	2.1283	64° 50′
25° 20′	.4279	.9038	.4734	2.1123	64° 40′
25° 30′	.4305	.9026	.4770	2.0965	64° 30′
25° 40′	.4331	.9013	.4806	2.0809	64° 20′
25° 50′	.4358	.9001	.4841	2.0655	64° 10′
26° 00′	.4384	.8988	.4877	2.0503	64° 00′
26° 10′	.4410	.8975	.4913	2.0353	63° 50′
26° 20′	.4436	.8962	.4950	2.0204	63° 40′
26° 30′	.4462	.8949	.4986	2.0057	63° 30′
26° 40′	.4488	.8936	.5022	1.9912	63° 20′
26° 50′	.4514	.8923	.5059	1.9768	63° 10′
27° 00′	.4540	.8910	.5095	1.9626	63° 00′
27° 10′	.4566	.8897	.5132	1.9486	62° 50′
27° 20′	.4592	.8884	.5169	1.9347	62° 40′
27° 30′	.4617	.8870	.5206	1.9210	62° 30′
27° 40′	.4643	.8857	.5243	1.9074	62° 20′
27° 50′	.4669	.8843	.5280	1.8940	62° 10′
28° 00′	.4695	.8829	.5317	1.8807	62° 00′
28° 10′	.4720	.8816	.5354	1.8676	61° 50′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
28° 20′	.4746	.8802	.5392	1.8546	61° 40′
28° 30′	.4772	.8788	.5430	1.8418	61° 30′
28° 40′	.4797	.8774	.5467	1.8291	61° 20′
28° 50′	.4823	.8760	.5505	1.8165	61° 10′
29° 00′	.4848	.8746	.5543	1.8040	61° 00′
29° 10′	.4874	.8732	.5581	1.7917	60° 50′
29° 20′	.4899	.8718	.5619	1.7796	60° 40′
29° 30′	.4924	.8704	.5658	1.7675	60° 30′
29° 40′	.4950	.8689	.5696	1.7556	60° 20′
29° 50′	.4975	.8675	.5735	1.7437	60° 10′
30° 00′	.5000	.8660	.5774	1.7321	60° 00′
30° 10′	.5025	.8646	.5812	1.7205	59° 50′
30° 20′	.5050	.8631	.5851	1.7090	59° 40′
30° 30′	.5075	.8616	.5890	1.6977	59° 30′
30° 40′	.5100	.8601	.5930	1.6864	59° 20′
30° 50′	.5125	.8587	.5969	1.6753	59° 10′
31° 00′	.5150	.8572	.6009	1.6643	59° 00′
31° 10′	.5175	.8557	.6048	1.6534	58° 50′
31° 20′	.5200	.8542	.6088	1.6426	58° 40′
31° 30′	.5225	.8526	.6128	1.6319	58° 30′
31° 40′	.5250	8511	.6168	1.6212	58° 20′
31° 50′	.5275	.8496	.6208	1.6107	58° 10′
32° 00′	.5299	.8480	.6249	1.6003	58° 00′
32° 10′	.5324	.8465	.6289	1.5900	57° 50′
32° 20′	.5348	.8450	.6330	1.5798	57° 40′
32° 30′	.5373	.8434	.6371	1.5697	57° 30′
32° 40′	.5398	.8418	.6412	1.5597	57° 20′
32° 50′	.5422	.8403	.6453	1.5497	57° 10′
33° 00′	.5446	.8387	.6494	1.5399	57° 00′
33° 10′	.5471	.8371	.6536	1 5301	56° 50′
33° 20′	.5495	.8355	.6577	1.5204	56° 40′
33° 30′	.5519	.8339	.6619	1.5108	56° 30′
33° 40′	.5544	.8323	.6661	1.5013	56° 20′
33° 50′	.5568	.8307	.6703	1.4919	56° 10′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
34° 00′	.5592	.8290	.6745	1.4826	56° 00′
34° 10′	.5616	.8274	.6787	1.4733	55° 50′
34° 20′	.5640	.8258	.6830	1.4641	55° 40′
34° 30′	.5664	.8241	.6873	1.4550	55° 30′
34° 40′	.5688	.8225	.6916	1.4460	55° 20′
34° 50′	.5712	.8208	.6959	1.4370	55° 10′
35° 00′	.5736	.8192	.7002	1.4281	55° 00′
35° 10′	.5760	.8175	.7046	1.4193	54° 50′
35° 20′	.5783	.8158	.7089	1.4106	54° 40′
35° 30′	.5807	.8141	.7133	1.4019	54° 30′
35° 40′	.5831	.8124	.7177	1.3934	54° 20′
35° 50′	.5854	.8107	.7221	1.3848	54° 10′
36° 00′	.5878	.8090	.7265	1.3764	54° 00′
36° 10′	.5901	.8073	.7310	1.3680	53° 50′
36° 20′	.5925	.8056	.7355	1.3597	53° 40′
36° 30′	.5948	.8039	.7400	1.3514	53° 30′
36° 40′	.5972	.8021	.7445	1.3432	53° 20′
36° 50′	.5995	.8004	.7490	1.3351	53° 10′
37° 00′	.6018	.7986	.7536	1.3270	53° 00′
37° 10′	.6041	.7969	.7581	1.3190	52° 50′
37° 20′	.6065	.7951	.7627	1.3111	52° 40′
37° 30′	.6088	.7934	.7673	1.3032	52° 30′
37° 40′	.6111	.7916	.7720	1.2954	52° 20′
37° 50′	.6134	.7898	.7766	1.2876	52° 10′
38° 00′	.6157	.7880	.7813	1.2799	52° 00′
38° 10′	.6180	.7862	.7860	1.2723	51° 50′
38° 20′	.6202	.7844	.7907	1.2647	51° 40′
38° 30′	.6225	.7826	.7954	1.2572	51° 30′
38° 40′	.6248	.7808	.8002	1.2497	51° 20′
38° 50′	.6271	.7790	.8050	1.2423	51° 10′
39° 00′	.6293	.7771	.8098	1.2349	51° 00′
39° 10′	.6316	.7753	.8146	1.2276	50° 50′
39° 20′	.6338	.7735	.8195	1.2203	50° 40′
39° 30′	.6361	.7716	.8243	1.2131	50° 30′
	cos	sin	cot	tan	Degrees

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS (*Continued*)

Degrees	sin	cos	tan	cot	
39° 40′	.6383	.7698	.8292	1.2059	50° 20′
39° 50′	.6406	.7679	.8342	1.1988	50° 10′
40° 00′	.6428	.7660	.8391	1.1918	50° 00′
40° 10′	.6450	.7642	.8441	1.1847	49° 50′
40° 20′	.6472	.7623	.8491	1.1778	49° 40′
40° 30′	.6494	.7604	.8541	1.1708	49° 30′
40° 40′	.6517	.7585	.8591	1.1640	49° 20′
40° 50′	.6539	.7566	.8642	1.1571	49° 10′
41° 00′	.6561	.7547	.8693	1.1504	49° 00′
41° 10′	.6583	.7528	.8744	1.1436	48° 50′
41° 20′	.6604	.7509	.8796	1.1369	48° 40′
41° 30′	.6626	.7490	.8847	1.1303	48° 30′
41° 40′	.6648	.7470	.8899	1.1237	48° 20′
41° 50′	.6670	.7451	.8952	1.1171	48° 10′
42° 00′	.6691	.7431	.9004	1.1106	48° 00′
42° 10′	.6713	.7412	.9057	1.1041	47° 50′
42° 20′	.6734	.7392	.9110	1.0977	47° 40′
42° 30′	.6756	.7373	.9163	1.0913	47° 30′
42° 40′	.6777	.7353	.9217	1.0850	47° 20′
42° 50′	.6799	.7333	.9271	1.0786	47° 10′
43° 00′	.6820	.7314	.9325	1.0724	47° 00′
43° 10′	.6841	.7294	.9380	1.0661	46° 50′
43° 20′	.6862	.7274	.9435	1.0599	46° 40′
43° 30′	.6884	.7254	.9490	1.0538	46° 30′
43° 40′	.6905	.7234	.9545	1.0477	46° 20′
43° 50′	.6926	.7214	.9601	1.0416	46° 10′
44° 00′	.6947	.7193	.9657	1.0355	46° 00′
44° 10′	.6967	.7173	.9713	1.0295	45° 50′
44° 20′	.6988	.7153	.9770	1.0235	45° 40′
44° 30′	.7009	.7133	.9827	1.0176	45° 30′
44° 40′	.7030	.7112	.9884	1.0117	45° 20′
44° 50′	.7050	.7092	.9942	1.0058	45° 10′
45° 00′	.7071	.7071	1.0000	1.0000	45° 00′
	cos	sin	cot	tan	Degrees

HYPERBOLIC SINES AND COSINES

n	cosh n	sinh n	n	cosh n	sinh n
0.00	1.0000	0.0000	2.05	3.9484	3.8196
0.05	1.0013	0.0500	2.10	4.1443	4.0219
0.10	1.0050	0.1002	2.15	4.3507	4.2342
0.15	1.0112	0.1506	2.20	4.5679	4.4571
0.20	1.0201	0.2013	2.25	4.7966	4.6912
0.25	1.0314	0.2526	2.30	5.0372	4.9369
0.30	1.0453	0.3045	2.35	5.2905	5.1952
0.35	1.0619	0.3572	2.40	5.5569	5.4662
0.40	1.0811	0.4108	2.45	5.8373	5.7510
0.45	1.1030	0.4653	2.50	6.1323	6.0502
0.50	1.1276	0.5211	2.55	6.4426	6.3645
0.55	1.1551	0.5782	2.60	6.7690	6.6947
0.60	1.1855	0.6367	2.65	7.1123	7.0417
0.65	1.2188	0.6967	2.70	7.4735	7.4063
0.70	1.2552	0.7586	2.75	7.8533	7.7894
0.75	1.2947	0.8223	2.80	8.2527	8.1919
0.80	1.3374	0.8881	2.85	8.6728	8.6150
0.85	1.3835	0.9561	2.90	9.1146	9.0596
0.90	1.4331	1.0265	2.95	9.5791	9.5268
0.95	1.4862	1.0995	3.00	10.0677	10.0179
1.00	1.5431	1.1752	3.05	10.5814	10.5340
1.05	1.6038	1.2539	3.10	11.1215	11.0765
1.10	1.6685	1.3356	3.15	11.6895	11.6466
1.15	1.7374	1.4208	3.20	12.2866	12.2459
1.20	1.8107	1.5097	3.25	12.9146	12.8758
1.25	1.8884	1.6019	3.30	13.5748	13.5379
1.30	1.9709	1.6984	3.35	14.2689	14.2338
1.35	2.0583	1.7991	3.40	14.9987	14.9654
1.40	2.1509	1.9043	3.45	15.7661	15.7343
1.45	2.2488	2.0143	3.50	16.5728	16.5426
1.50	2.3524	2.1293	3.55	17.4210	17.3923
1.55	2.4619	2.2496	3.60	18.3128	18.2855
1.60	2.5775	2.3757	3.65	19.2503	19.2243
1.65	2.6995	2.5075	3.70	20.2360	20.2113
1.70	2.8283	2.6456	3.75	21.2723	21.2488
1.75	2.9642	2.7904	3.80	22.3618	22.3394
1.80	3.1075	2.9422	3.85	23.5072	23.4859
1.85	3.2583	3.1013	3.90	24.7113	24.6911
1.90	3.4177	3.2682	3.95	25.9773	25.9581
1.95	3.5855	3.4432	4.00	27.3082	27.2899
2.00	3.7622	3.6269	....	......	......

HYPERBOLIC TANGENTS AND COTANGENTS

n	tanh n	coth n	n	tanh n	coth n
0.00	0.00000	∞	....	....	......
0.05	0.04996	20.017	2.05	0.96740	1.0337
0.10	0.09967	10.033	2.10	0.97045	1.0304
0.15	0.14889	6.7166	2.15	0.97323	1.0275
0.20	0.19738	5.0665	2.20	0.97574	1.0249
0.25	0.24492	4.0830	2.25	0.97803	1.0225
0.30	0.29131	3.4327	2.30	0.98010	1.0203
0.35	0.33638	2.9729	2.35	0.98197	1.0184
0.40	0.37995	2.6319	2.40	0.98367	1.0166
0.45	0.42190	2.3702	2.45	0.98522	1.0150
0.50	0.46212	2.1640	2.50	0.98661	1.0136
0.55	0.50052	1.9979	2.55	0.98788	1.0123
0.60	0.53705	1.8620	2.60	0.98903	1.0111
0.65	0.57167	1.7493	2.65	0.99007	1.0100
0.70	0.60437	1.6546	2.70	0.99101	1.0091
0.75	0.63515	1.5744	2.75	0.99186	1.0082
0.80	0.66404	1.5059	2.80	0.99263	1.0074
0.85	0.69107	1.4470	2.85	0.99333	1.0067
0.90	0.71630	1.3961	2.90	0.99396	1.0061
0.95	0.73978	1.3517	2.95	0.99454	1.0055
1.00	0.76159	1.3130	3.00	0.99505	1.0050
1.05	0.78181	1.2791	3.0	0.99505	1.0050
1.10	0.80050	1.2492	3.1	0.99595	1.0041
1.15	0.81775	1.2229	3.2	0.99668	1.0033
1.20	0.83365	1.1995	3.3	0.99728	1.0027
1.25	0.84828	1.1789	3.4	0.99777	1.0022
1.30	0.86172	1.1605	3.5	0.99818	1.0018
1.35	0.87405	1.1441	3.6	0.99851	1.0015
1.40	0.88535	1.1295	3.7	0.99878	1.0012
1.45	0.89569	1.1165	3.8	0.99900	1.0010
1.50	0.90515	1.1048	3.9	0.99918	1.0008
1.55	0.91379	1.0943	4.0	0.99933	1.0007
1.60	0.92167	1.0850	4.1	0.99945	1.0005
1.65	0.92886	1.0766	4.2	0.99955	1.0004
1.70	0.93541	1.0691	4.3	0.99963	1.0004
1.75	0.94138	1.0623	4.4	0.99970	1.0003
1.80	0.94681	1.0562	4.5	0.99975	1.0002
1.85	0.95175	1.0507	4.6	0.99980	1.0002
1.90	0.95624	1.0458	4.7	0.99983	1.0002
1.95	0.96032	1.0413	4.8	0.99986	1.0001
2.00	0.96403	1.0373	4.9	0.99989	1.0001

Numerical Constants

$$\pi = 3.141\ 592\ 654$$

$$\log_{10} \pi = 0.497\ 149\ 873$$

$$\frac{1}{\pi} = 0.318\ 309\ 886$$

$$\pi^2 = 9.869\ 604\ 401$$

$$\sqrt{\pi} = 1.772\ 453\ 851$$

$$\frac{1}{\sqrt{\pi}} = 0.564\ 189\ 583$$

$$\sqrt{\frac{\pi}{2}} = 1.253\ 314\ 137$$

$$\sqrt{\frac{2}{\pi}} = 0.797\ 884\ 561$$

$$e = 2.718\ 281\ 828$$

$$\frac{1}{e} = 0.367\ 879\ 441$$

$$\log_{10} e = 0.434\ 294\ 482$$

$$\log_e 10 = 2.302\ 585\ 093$$

$$\log_{10} \log_{10} e = 9.637\ 784\ 311$$

$$\log_e \pi = 1.144\ 729\ 886$$

$$\log_e 2 = 0.693\ 147\ 181$$

$$\log_{10} 2 = 0.301\ 029\ 996$$

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